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Elin Gawell: Rings of arithmetic functions with regular convolutions

Sammanfattning

An arithmetric function is a function from the natural numbers to the complex numbers. The term regular convolution was introduced by Narkiewicz in 1963. A regular convolution, $*$, is defined by $f*g(n) = \sum_{dinA(n)} f(d)g(n/d)$ where $A(n)$ is a set of divisors to the natural number n, such that the ring of arithmetic functions with this convolution as multiplication is

- commutative
- associative
- has the unit element, $e_1(n) = 1$ if $n = 1$, 0 otherwise
- preserves multiplicativity
- the inverse function of $f(n) = 1$ for all n, called the "Möbius-function", m, defined by $f * m = e_1$, take only the values 0 and -1 for prime powers

The most well-known examples of regular convolutions are the Dirichlet convolution and the unitary convolution. Each convolutions is determined by a family of partitions of the natural numbers into arithmetic progressions (which all contains 0). By regarding these progressions as incidence algebras we find the previously undescribed ternary convolution.

We find an explicit formula for the inverse of invertible elements in the ring with ternary convolution which accually works in rings with any regular convolution.

We also show that there exist only one regular convolution on the ring of arithmetic functions such that $f(n) = 0$ if n not square-free.