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Matilda Kapro: Hall's Marriage Theorem

Sammanfattning

Hall's Marriage Theorem, due to Philip Hall 1935, is an important result in graph theory. It gives necessary and sufficient conditions for a perfect matching to exist between the two sets of vertices of a bipartite graph G, i.e. a set of disjoint edges containing all vertices of G. In this paper I describe this theorem and give different ways to prove it, using different theorems in graph theory such as König's, Berge's, and The Max-Flow Min-Cut theorems. I then present the Stable Marriage Problem whose solution is a consequence of Hall's Theorem and can be modelled as a perfect matching problem. In an implementation of the Stable Marriage Problem, each of n men and n women ranks the members of the opposite sex in order of preference, and one has to find the best possible way of marrying them. It is well known that at least one stable matching exists for every Stable Marriage Problem as we will see in this paper. I also provide an algorithm for finding a stable matching, namely the Gale-Shapley algorithm which is a well known procedure for solving the classical Stable Marriage Problem.