EXAMENSARBETEN I MATEMATIK

MATEMATISKA INSTITUTIONEN, STOCKHOLMS UNIVERSITET

Increased Wealth through Mathematics?

av

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2008 - No 8

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Examensarbete i matematik 15 högskolepoäng, påbyggnadskurs

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2008

Abstract

In this thesis we will construct a number of different fund portfolios with different objectives and constraints, using portfolio theory. All of the constructed portfolios are possible to create in the Swedish premium pension system and we will follow their progress while comparing them to each other and to the default choice provided by the Seventh AP fund. Our goal is both to check that the theory is applicable to funds and if our findings are consistent with the predicted outcome. Furthermore it is of great interest to see if there is any substantial economic gain in applying portfolio theory to the fund selection process.

Contents

1 Introduction

In June 1998, the Swedish Parliament decided that a new pension system was to be introduced – the National Pension Scheme. The key reason for changing the pension system was that the existing system from 1960, comprising the National Basic Pension and the National Supplementary Pension, was no longer stable. It was concluded that the old system would eventually result in an imbalance in the relationship between the amounts paid into the system and the pensions paid out. In order to cover the pension payments, it would be necessary to increase charges during periods of low economic growth and this would lead to insecurity for all parties; both those already receiving a pension and those who were still working.

It was also referred to other pressures on the system in the form of major changes in the population structure after 2000. There would be an increase in the number of retired persons, resulting in increased pension payments, while the proportion of the population that would need to support this group would decrease. In addition, due to a higher life expectancy, pensions would also need to be paid out over a longer period.

1.1 The present pension system

The new Swedish pension system is a mixture of a fund-based system, the premium pension, and a distribution-based system, a guaranteed income-based pension. Everyone who earns an income and who pays tax in Sweden also earns pension rights. The pension charge corresponds to 18,5% of earned income and other pensionable amounts up to a certain ceiling, of which 16 percentage points are used to finance the pensions paid out to today's beneficiaries. The pension rights earned by each individual during a specific year are registered and are then added over the years. The total is also indexed to keep pace with wage increases in the country.

The remaining 2,5 percentage points are used to fund the premium pension. The money reserved for the premium pension may be invested in funds by the pension-savers, who personally assume full financial responsibility for these investments. The size of the National Pension depends on the total amount the individual earns throughout his/her entire working life, when he/she decides to begin claiming the pension, socio-economic developments and the success of the pension-saver's fund investments. A so called guaranteed pension is the basic financial cover for those individuals who have not earned a sufficient amount of either income based pension or premium pension.

In addition to the national pension many individuals are entitled to some form of negotiated pension from their employer, and private pension schemes are also becoming very popular and are to some extent encouraged by the government due to the possibility of tax deduction up to a certain amount each year.

1.2 PPMs role in this system

The Premium Pension Authority, (PPM)
PPMs mission is to manage the premium pension system in a sound, cost-effective and legally correct manner. The authority must also provide a good service to the pension-savers by providing them with information about the premium pension system so that they can make well-informed choices. PPM is like a market of funds where PPM invests money in the funds that the pension savers have chosen.

Each year, everyone who has earned pension rights receives a joint statement from their Local Social Insurance Office (Försäkringskassan) and PPM, containing personal information about their future National Pension. The statement includes information about the size of the pension rights they have earned, a forecast for the National Pension they can expect to receive under current circumstances and information about any changes in the value of their chosen funds.

PPM is financed through a charge made on the pension-savers' accounts. During the initial establishment period, however, financing takes the form of a combination of these charges and loans from the National Debt Office. These loans are to be repaid in stages.

1.3 The Seventh AP Fund and the Premium savings fund

The Seventh AP Fund (AP7) was set up in connection with the introduction of Swedens new pension system. Their sole task is to manage the premium pension for the people of Sweden. All the rules and regulations that apply to all private fund managers, who are participating with their fund in PPMs range, also apply to AP7. The premiums of those who preferred not to choose any funds in the PPM system, will be invested in the Premium savings fund, but if they ever make an active choice, their capital can never be reinvested in this fund. The fact that AP7 is managing the Premium savings fund is by no means any guarantee for the returns of the fund. They are as dependent as all the other fund managers, of the progress of the financial markets and the skills of their traders to generate growth to their investors.

The goal of the Premium savings fund is that the return of the fund in any five-year period should correspond to at least the average return of all the funds included in the PPM system of the same period. The Premium savings fund also has one of the lowest fees of all the funds. The gross fee is 0,5% and PPM requires a discount that is reinvested into the investors account leading to a net fee of 0,16%.

Most of the capital received by the Premium savings fund, currently 82%, is invested in shares listed on a stock market. Most of these investments (65%) are foreign equities, among them American, European and Asian shares, and 17% are invested in Sweden. Listed equities are managed in such a way that they track the stock market index in each country, which is known as passive management. However, around 40% is managed actively with the object of earning a higher return than the benchmark index. This means that local portfolio managers in each country buy and sell shares regularly in order to maximise the return.

The Premium savings fund also invests in inflation-linked bonds. Securities of this kind are issued by Sweden, among other borrowers, and provide a guaranteed return as well as protecting the investor against losses from inflation. The fund also invests in alternative investments such as hedge funds and private equity funds.

1.4 Who enters this system?

Last year (2007), everyone who has had an annual income of more than 17 047 SEK, and who had not yet been entered into the system, was asked to make a choice of which funds to place their premium pension in. They were given the option to invest their money in up to five different funds. The number of available funds is currently just less than 800, managed by 80-90 different fund managers.

The result of 2006 years selection is that out of 113 604 persons qualified to make their first selection, only 8% made an active choice. If we as an alternative look at how the money was distributed among the funds, we find that out of nearly 147 million SEK, more than 135 million was given to be managed by AP7 in their Premium savings fund.

2 Portfolio theory

In 1952, Harry Markowitz published a paper entitled “Portfolio Selection”, where he showed how to create a frontier of portfolios, such that each of them had the greatest possible expected rate of return given their level of risk. Investors before then knew intuitively that it was smart to diversify, *don’t put all your eggs in one basket*. Markowitz was among the first to attempt to quantify risk and demonstrate quantitatively why and how portfolio diversification works to reduce risk for investors.

The process for establishing an optimal (or efficient) portfolio generally uses historical measures for returns, risk and correlation coefficients for each asset to be used in the portfolio. Historical measures are used as a proxy for expected future returns which may or may not be true, particularly over the short term. History is a better indicator if used over the long term.

In order to create and calculate any portfolios we need to agree on certain financial definitions:

- Return
  Return is a term that is understood by most investors. Total return is a measure of the combined income and capital gain from an investment. This is usually expressed as a percentage which may be annualised over a number of years or represent a single period.

- Risk
  While there are many types of risk and different methods of measuring them, the standard deviation of (historical) returns are probably the most common measure of the risk of listed securities and portfolios. It measures the variability of returns and the higher the standard deviation, the more uncertain the outcome over any period. Standard deviation is very useful because it makes it possible to compare the risk between different types of investment, for example shares against bonds.

- Correlation
  Correlation is computed into what is known as the correlation coefficient, which ranges between -1 and +1. Perfect positive correlation (a correlation coefficient of +1) implies that as one security moves, either up or down, the other security will move in the same direction. Alternatively, perfect negative correlation means that if one security moves in either direction the security that is perfectly negatively correlated will move by an equal amount in the opposite direction. If the correlation is 0, the movements of the securities is said to have no correlation, it is completely random. If one security moves up or down

there is as good a chance that the other will move either up or down, the way in which they move is totally random. In real life however there will not likely be any perfectly correlated securities, rather there will exist securities with some degree of correlation. For example, the performance of two stocks within the same industry is strongly positively correlated although it may not be exactly +1.

2.1 General constraints

Below are a number of constraints that affect both the estimated return and the risk in our portfolios and in this case all of these are having a negative influence on the expected return of our portfolio.

2.1.1 No short selling

When you are a trader on a stock market, the possibility of short selling is always an option, which basically means that you are selling a asset that you don't own, with the expectation that the price will fall in order to be able to buy it back later at a lower price, thus generating a profit on a declining market. Once the asset is sold, its weight in the traders portfolio is negative, meaning it is a dept he has to fulfil in time. This option is not available for a person buying or selling funds in contrast to stocks, for example. That is why we cannot accept any funds that generate negative weights in our portfolio.

2.1.2 Maximum of five funds

When we want to start buying some of the funds that are presented to us, we must limit ourselves to choose only five different funds in our holding. This means that if we in our calculations find out that the optimal portfolio with the current constraint would require a portfolio consisting of more than five funds, we must reduce the number of funds. This will, where appropriate, be done in the following way. The funds with least significant weight will be eliminated from our portfolio and the five most significant will be proportionally increased to fill the void.

2.1.3 Integers

We have one final restriction to take into consideration before we can start our fund trading, namely the fact that we have to write our diversification percentage as integers, meaning that we have to use rounded numbers. This also renders the possibility that the sum of the rounded percentage might not be exactly 100%. Any deviations from 100% will in this case be corrected in the percentage of the most significant fund in our portfolio, also leading to a small difference from the optimal fund composition.

2.2 Presenting the funds

All the available funds have been divided into the following four different categories by PPM:

- Interest funds, which generally generate a limited, but stable return with very low risk. In this category we will find our substitute to risk free interest rate.

- Share funds that mainly invest in company shares. These funds can also be specialised in certain business sectors such as IT or telecom or they can be devoted to different geographical areas such as Far East or Sweden. Some of these funds also separate between investing in shares from companies of different size such as large, mid and small cap. Share funds that invest in other countries than Sweden are also exposed to fluctuations in currency exchange rates. In this category you will find the most volatile funds.

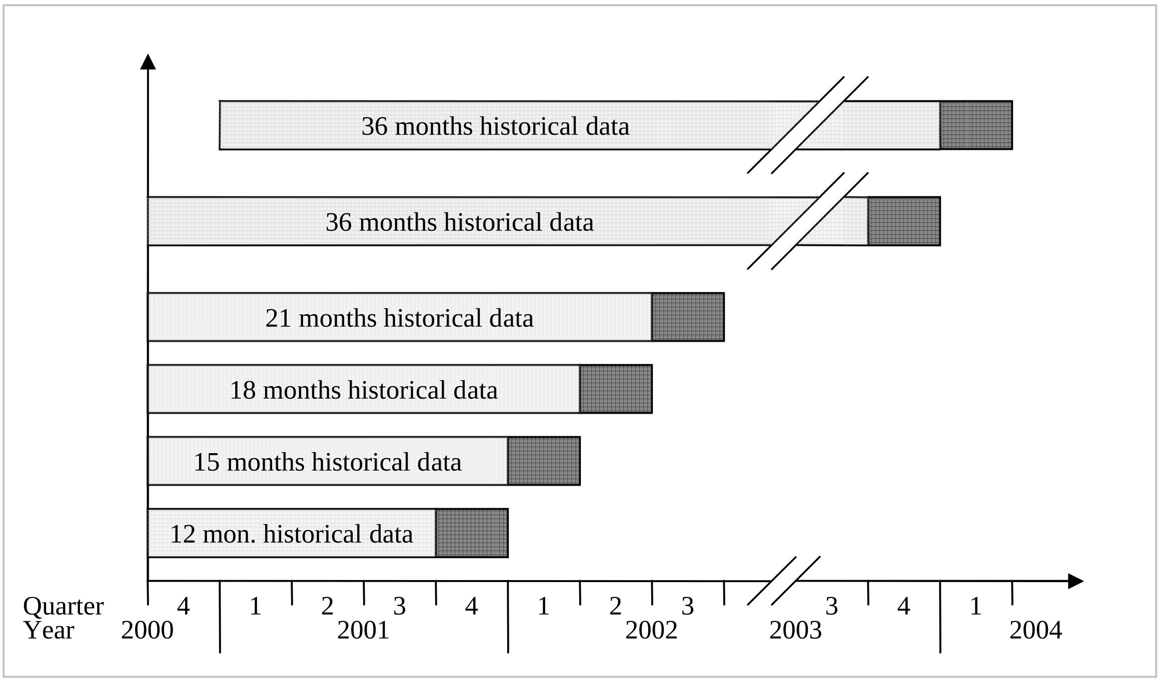
- Mixed funds contain both shares and interest bearing securities and present an easy alternative to creating your own mix between the two, but since we aim to optimize our own portfolios, this fund type will be disregarded.

- Generation funds are similar to the previous category, with the difference that these funds change the relation of interest securities vs. shares, over time. All with the intention to secure the capital invested in them as retirement approaches. Since our objective is to control our risk exposure ourselves, this category also will be discarded.

2.3 How much historical data and how far ahead to prognosticate?

The problem with using historical data to make any kind of prediction of the future is that not only do you need a lot of data; you also need data to be recent, and you are faced with an inconsistency that is hard to act in accordance with. But we are still going to make several fund portfolios that we plan to hold for the coming quarter of a year and the least amount of historical data that we need for making such a prognosis with any certainty is at least a year. We restrict the amount of historical data needed to make our quarterly portfolios to 36 months. This means that the first quarter that we can prognosticate is Q4 2001, since the available data is dated from 2000-09-21. Not all funds that are available today, have historical data that reaches so far back, but they will make their appearance in our calculations in pace with the proceedings of our prognosis.

During the first twelve month period we assume that our capital is invested in the Premium savings fund.



The dark grey area shows the period where we apply our prognosis and the lighter grey area shows how much historical data we use in order to create that prognosis.

2.3.1 Weekly data

Because the share funds that are available to us, is of both national and international types, one understands that it might not be so straightforward to find any correlation between market events taking place at the opening of the Tokyo stock market early one day, to some other market events taking place just before closing time in New York later that same day with a time span of nearly 22 hours later. Therefore we are not using daily quotes in our calculations but weekly quotes instead, which should give us a better correlation between the different funds.

2.3.2 Difference between buy and sell rates

Since some of the funds presented by PPM is traded in another currency than SEK, there is a difference between the funds buy and sell rates. We will use an average of these two rates in our calculations in order to simplify them.

2.3.3 Only share funds

We will only use the share funds in our calculations to produce our different portfolios. Since the goal of this exercise is to find several optimal portfolios that we ourselves can combine with the risk free interest rate, in order to adjust our overall risk.

2.3.4 Too many funds - how to select the top 30

The share fund type has by far the largest amount of funds in the assortment presented to us. That is why we need to restrict our choice of funds in order to keep our calculations manageable. More on the different restrictions we use later.

2.4 Portfolios

The different portfolios that we are going to construct are the following:

- Minimum variance portfolio. The portfolio consisting of only share funds that has the least variance possible, given the previously mentioned general constraints. The funds in this portfolio are chosen only because of their effect on the variance regardless of their estimated return. Instead of using all of the nearly 600 available share funds in our calculation, we limit ourselves to start our computation with the 30 funds that has the least variance during the measurement period.

- Maximum growth portfolio. In the design of this portfolio we simply try to get the highest possible growth and in order to calculate this portfolio without using all share funds, we use the 30 share funds that have got the highest Sharpe ratio.

- Markowitz portfolio. This is also known as the tangency portfolio. It is the portfolio with the highest possible Sharpe ratio. That is the highest ratio between excess return over the risk free interest and the volatility of the portfolio. Here it comes naturally to select the top 30 funds with the highest Sharpe Ratio to facilitate our calculations.

- AP7-replica. This portfolio will be designed to match the volatility of the Premium savings fund. For each quarter we will find the standard deviation of the returns of the Premium savings fund during our measurement period and find an optimal portfolio that equals this volatility, although this portfolio might not consist of share funds alone. In the case that the volatility we are trying to match is lower than the volatility of the Markowitz portfolio, we get a higher expected return on a portfolio consisting of a linear combination of the Markowitz portfolio and the risk free interest. It's not that apparent which funds to select to avoid having to use all share funds in the calculation, but we use the top 30 funds with the highest Sharpe index here as well.

- Bull portfolio. Here we simply choose the single fund with the highest return during our measurement period, to be our choice of investment for the coming quarter and we will not take any risk into consideration. This portfolio will be part of our comparison only to serve as a match against the maximum growth portfolio.

These five portfolios will be followed and recalculated each quarter and we will also keep track of the AP7s Premium savings fund, since that is the default choice and by far the single biggest participant in the premium pension arrangement.

2.5 Mathematical definitions

Before we start to construct our different portfolios we need the following definitions. While we consider a portfolio consisting of m assets, we let the asset prices at time t be denoted by $S_1(t),\dots,S_m(t)$, and let $V_j(t)$ be the value of asset j at time t, $j=1,\dots,m$; $V_j(t)=a_jS_j(t)$, where a_j is the amount of asset j in the portfolio. The total value of the portfolio at time t is given by

$$P(t)=V_1(t)+\dots+\mathrm{V}_m(\mathrm{t}),$$

and the asset j has the weight

$$v_j(t)=V_j(t)/P(t)$$

in the portfolio.

The return of the portfolio in the time interval $(t,\,t+\partial t)$ is given by

$$R_P(t,t+\partial t)=\frac{P(t,t+\partial t)-P(t)}{P(t)}=\sum_{j=1}^{m}v_j(t)R_j(t,t+\partial t),$$

where

$$R_j(t, t+\partial t) = \frac{S_j(t, t+\partial t) - S_j(t)}{S_j(t)}$$

is the return of asset *j* during the time interval. To compute the estimated return $E\lfloor R_j \rfloor$ of asset *j*, we use historical sample data. The return of the portfolio thus has the following variance

$$\begin{aligned} &Var(R_P(t, t+\partial t)) = \\ &= \sum_{j=1}^{m}\sum_{k=1}^{m} v_j(t) v_k(t) Cov(R_j(t, t+\partial t), R_k(t, t+\partial t)) = \\ &= \mathbf{v}(t)^T Q_{\partial t} \mathbf{v}(t) \end{aligned}$$

where $\mathbf{v}(t) = (v_1(t), \ldots, v_m(t))^T$ and Q is the covariance matrix.

2.5.1 Risk free interest

In order to construct several of these portfolios we need a substitute for the risk free interest rate and in our case it is convenient to choose the interest rate fund with the smallest standard deviation during the measurement period. The results of these selections are as follows.

Period	Fund which substitutes the risk free interest rate	Estimated weekly return
2001 Q4	Danske Fonder - Sverige Likviditet	0,00076
2002 Q1	Moderna Fonder - Sverige Ränta	0,00077
2002 Q2	Moderna Fonder - Sverige Ränta	0,00074
2002 Q3	Moderna Fonder - Sverige Ränta	0,00075
2002 Q4	Moderna Fonder - Sverige Ränta	0,00076
2003 Q1	Moderna Fonder - Sverige Ränta	0,00078
2003 Q2	Moderna Fonder - Sverige Ränta	0,00076
2003 Q3	Moderna Fonder - Sverige Ränta	0,00076
2003 Q4	Moderna Fonder - Sverige Ränta	0,00073
2004 Q1	Moderna Fonder - Sverige Ränta	0,00071
2004 Q2	Moderna Fonder - Sverige Ränta	0,00069
2004 Q3	Moderna Fonder - Sverige Ränta	0,00067
2004 Q4	Moderna Fonder - Sverige Ränta	0,00061
2005 Q1	Moderna Fonder - Sverige Ränta	0,00059
2005 Q2	Moderna Fonder - Sverige Ränta	0,00056
2005 Q3	Handelsbanken Fonder - Lux Ränta	0,00051
2005 Q4	Moderna Fonder - Sverige Ränta	0,00048
2006 Q1	Moderna Fonder - Sverige Ränta	0,00042
2006 Q2	Moderna Fonder - Sverige Ränta	0,00039
2006 Q3	Moderna Fonder - Sverige Ränta	0,00035
2006 Q4	Handelsbanken Fonder - Lux Ränta	0,00035
2007 Q1	Handelsbanken Fonder - Lux Ränta	0,00034
2007 Q2	Handelsbanken Fonder - Lux Ränta	0,00034
2007 Q3	Moderna Fonder - Sverige Ränta	0,00035
2007 Q4	Handelsbanken Fonder - Lux Ränta	0,00037

3 Minimum variance portfolio

The minimum variance portfolio is the choice of the cautious investor, but even so, we decide to create this risk avert portfolio out of share funds only. By selecting the 30 share funds that has the lowest standard deviation of returns during our measurement period, we get a manageable subset of all the share funds at our disposal and it is out of this subset that we carry out our calculations.

To construct the portfolio of share funds, that has the least variance we need to find the weights $\mathbf{v}$ that minimizes

$$\frac{1}{2}\mathbf{v}^T Q\mathbf{v} = \frac{1}{2}\sum_i \sum_j v_i \sigma_{i,j} v_j$$

subject to the constraint $\sum_i v_i = 1$.
Using Lagrange multipliers we get the following expression

$$L(\mathbf{v},\lambda) = \sum_i \sum_j v_i \sigma_{i,j} v_j + \lambda\left[\sum_i v_i - 1\right].$$

A necessary condition for a minimum is that all first derivatives are 0, which gives

$$\frac{\partial L}{\partial \mathbf{v}} = \sum_j \sigma_{i,j} v_j - \lambda = 0, \quad i = 1,\ldots,m, \quad \sum_j v_j = 1.$$

The above expression can also be written as

$$Q\,\mathbf{v} = \lambda \mathbf{1}, \quad \mathbf{1v} = 1$$

where $\mathbf{1} = (1,\ldots,1)^T$, and we get $\mathbf{v} = \lambda Q^{-1}\mathbf{1}$ and the constraint gives $\lambda \mathbf{1}^T \cdot Q^{-1}\mathbf{1} = 1$. The minimal variance is

$$\begin{aligned}
\mathbf{v}^T Q\,\mathbf{v} &= \left(\lambda Q^{-1}\mathbf{1}\right)^T Q\left(\lambda Q^{-1}\mathbf{1}\right) = \\
&= \lambda^2 \mathbf{1}^T Q^{-1} Q Q^{-1}\mathbf{1} = \\
&= \lambda^2 \mathbf{1}^T Q^{-1}\mathbf{1} = \left[\lambda = 1/\mathbf{1}^T Q^{-1}\mathbf{1}\right] = \\
&= 1/\mathbf{1}^T Q^{-1}\mathbf{1}.
\end{aligned}$$

So we have that the minimum variance portfolio has variance

$$\sigma^2_{\min} = \lambda = 1 \big/ \mathbf{1}^T Q^{-1}\mathbf{1}$$

and the weights

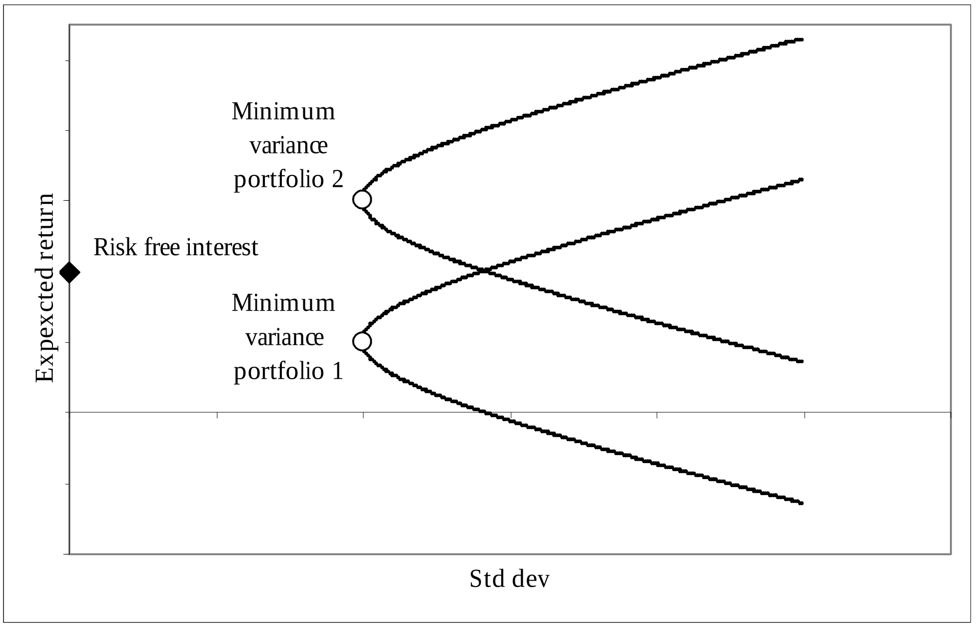
$$\mathbf{v}_{\min} = \lambda Q^{-1}\mathbf{1} = \sigma^2_{\min} Q^{-1}\mathbf{1}.$$

This then gives that the expected return of the minimum variance portfolio is

$$\mu_{\min} = \vec{\mu}\cdot\mathbf{v}_{\min} = \sigma^2_{\min}\mathbf{1}\cdot Q^{-1}\vec{\mu}\,.$$

3.1 Two different scenarios

When calculating the minimum variance portfolio we might get two different results for our expected return. Either we get the expected return that is below the return of our risk free alternative, as in the case of portfolio 1 in the graph below. In this case we simply choose to keep our money in cash. (Investing the money in the fund that represents the risk free interest rate will from here on be noted as keeping it in cash.) Or we might get the scenario as in our example with the second portfolio in the same graph. Here the expected return of our portfolio is higher than the risk free interest and we choose to invest in the portfolio.



3.2 Empirical results

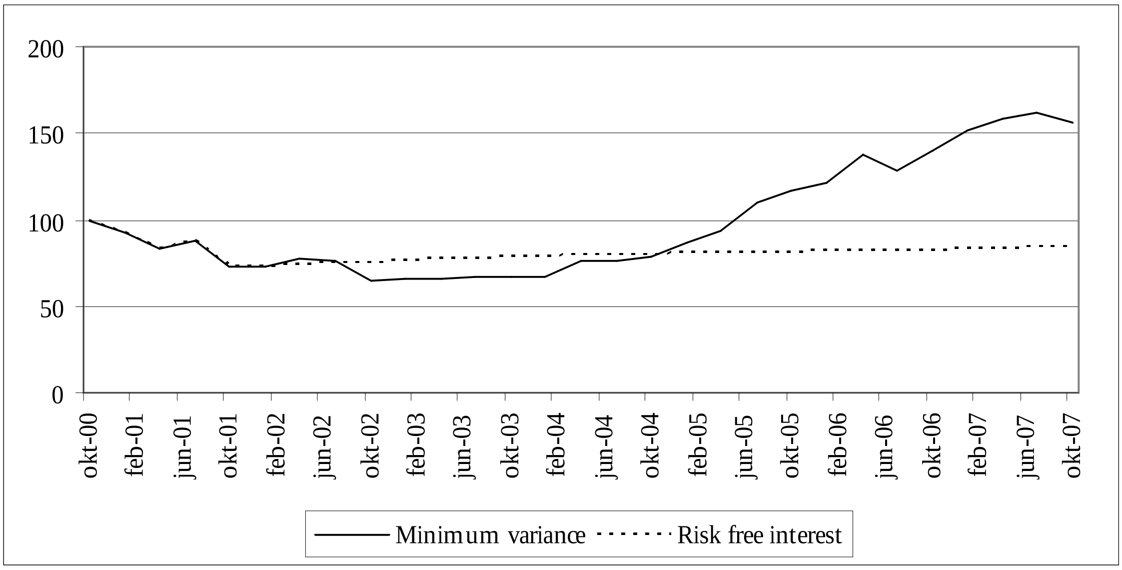
Following the previous calculations, these are the fund combinations that generate the minimum variance portfolio for the first period. Each estimate is calculated using weekly data.

2001 Q4	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00114	0,0138	0,72
Morgan Stanley SICAV European Equity Fund	-0,00107	0,0160	0,03
SGAM Fund Equities Switzerland	-0,00295	0,0188	0,08
Danske Fonder Global Index	-0,00375	0,0199	0,07
Carnegie Fund - WorldWide Sub-Fund	-0,00730	0,0224	0,10
Minimum variance portfolio	-0,00044	0,01288	1,00
Risk free interest			
Danske Fonder - Sverige Likviditet	0,00076		

In this case the expected return of the portfolio is smaller than the risk free interest so we choose to keep our money in the bank for this period. The resulting fund portfolios of all periods can be found in Appendix A1. We only invest our money in the minimum variance portfolio when its expected return is higher than the corresponding risk free interest rate for the same period. The subsequent table gives a description of how we choose to invest by following the strategy of minimizing our exposure to risk.

Period	Risk free interest rate	Exp return from minimum variance portfolio	Resulting investment choice
2001 Q4	0,00076	-0,00044	Cash
2002 Q1	0,00077	0,00086	MinVar portfolio
2002 Q2	0,00074	0,00132	MinVar portfolio
2002 Q3	0,00075	0,00106	MinVar portfolio
2002 Q4	0,00076	-0,00072	Cash
2003 Q1	0,00078	-0,00002	Cash
2003 Q2	0,00076	-0,00058	Cash
2003 Q3	0,00076	0,00016	Cash
2003 Q4	0,00073	0,00052	Cash
2004 Q1	0,00071	0,00098	MinVar portfolio
2004 Q2	0,00069	0,00126	MinVar portfolio
2004 Q3	0,00068	0,00127	MinVar portfolio
2004 Q4	0,00061	0,00225	MinVar portfolio
2005 Q1	0,00059	0,00251	MinVar portfolio
2005 Q2	0,00057	0,00286	MinVar portfolio
2005 Q3	0,00051	0,00393	MinVar portfolio
2005 Q4	0,00048	0,00477	MinVar portfolio
2006 Q1	0,00042	0,00437	MinVar portfolio
2006 Q2	0,00039	0,00516	MinVar portfolio
2006 Q3	0,00035	0,00308	MinVar portfolio
2006 Q4	0,00035	0,00373	MinVar portfolio
2007 Q1	0,00034	0,00352	MinVar portfolio
2007 Q2	0,00034	0,00266	MinVar portfolio
2007 Q3	0,00034	0,00279	MinVar portfolio
2007 Q4	0,00037	0,00180	MinVar portfolio

The result of this strategy used during the periods Q4 2001 to Q4 2007 is that the minimum variance portfolio would have generated a profit of 160,8% while the risk free interest would have given the return of 15,9% in the same period. Remember that we chose to keep our money uninvested the first year, while gathering data to perform our analysis, so the first year our money was invested in the default alternative, the Premium savings fund managed by AP7. Unfortunately that fund lost 26,8% during its first year, so our strategies got of on a poor start. The graph shows the development of 100 SEK. invested at the start of October 2000. In the graph we present both the risk free alternative and the minimum variance portfolio.



4 Growth portfolios

From here on we shall consider portfolios consisting of both share funds and cash, which in our case will be replaced by the interest fund with the lowest volatility of each period. This funds estimated return will then be our substitute for the risk free interest, r_f. First of all we need to expand our definitions a bit and we start with the growth of an asset in the interval (t_1, t_2) as $G(t_1, t_2)$ and define it as

$$G(t_1, t_2) = \ln\left(\frac{S(t_2)}{S(t_1)}\right),$$

and we recall that the return of an asset in the time interval (t_1, t_2) is

$$R(t_1, t_2) = \frac{S(t_2) - S(t_1)}{S(t_1)}.$$

The relation between the two definitions above is

$$R(t_1, t_2) = e^{G(t_1, t_2)} - 1 \text{ or } G(t_1, t_2) = \ln(1 + R(t_1, t_2))$$

We also define the drift $\upsilon(t)$, to be the expected growth of an asset in the interval $(0, t)$, that is

$$\upsilon(t) = E[G(0,t)]$$

To further help us in our effort to construct rewarding portfolios, we define the auxiliary portfolio to be the portfolio with the weights.

$$\mathbf{v}_{aux} = Q^{-1}(\vec{\mu} - \mu_{\min}\mathbf{1}).$$

Here $\mu_{\min}$ is the expected return of the minimum variance portfolio. We will now see that the weights of the auxiliary portfolio sum to zero.

$$\begin{aligned}
\mathbf{1}\cdot\mathbf{v}_{aux} &= \mathbf{1}\cdot Q^{-1}(\vec{\mu} - \mu_{\min}\mathbf{1}) = \\
&= \mathbf{1}\cdot Q^{-1}\,\vec{\mu} - \mu_{\min}\mathbf{1}\cdot Q^{-1}\,\mathbf{1} = \\
&= \mathbf{1}\cdot Q^{-1}\,\vec{\mu} - \sigma_{\min}^2\mathbf{1}\cdot Q^{-1}\,\vec{\mu}\,\mathbf{1}\,Q^{-1}\mathbf{1} = \\
&= \mathbf{1}\cdot Q^{-1}\,\vec{\mu} - \sigma_{\min}^2\mathbf{1}\cdot Q^{-1}\,\vec{\mu}/\sigma_{\min}^2 = \\
&= \mathbf{1}\cdot Q^{-1}\,\vec{\mu} - \mathbf{1}\cdot Q^{-1}\,\vec{\mu} = 0.
\end{aligned}$$

4.1 Introducing cash

Let $\mathbf{v} = (v_1,...,v_m)^T$ and $\vec{\mu} = (\mu_1,...,\mu_m)^T$ denote the weights and the expected returns of the share funds in our portfolio. The weight of cash then equals $1 - v_1 - ... - v_m$. The expected return of portfolio then is

$$\begin{aligned}\mu_P &= r_f(1 - v_1 - ... - v_m) + \mu_1 v_1 + ... + \mu_m v_m = \\ &= r_f + (\mu_1 - r_f)v_1 + ... + (\mu_m - r_f)v_m = \\ &= r_f + (\vec{\mu} - r_f\,\mathbf{1})^T\,\mathbf{v}\end{aligned}$$

We now assume that the growth of the portfolio is given by

$$\upsilon_P = \mu_P - \frac{1}{2}\sigma_P^2 .$$

where

$$\sigma_P^2 = \mathbf{v}^T Q\,\mathbf{v}$$

is the variance of the portfolio. We rewrite the expression of the growth of the portfolio as follows

$$\upsilon_P = r_f + (\vec{\mu} - r_f\,\mathbf{1})^T\,\mathbf{v} - \frac{1}{2}\mathbf{v}^T Q\,\mathbf{v}$$

Maximizing the expected growth means

$$\begin{aligned}\frac{\partial \upsilon_p}{\partial \mathbf{v}} &= \vec{\mu} - r_f\,\mathbf{1} - Q\,\mathbf{v} = 0 \quad \Leftrightarrow \\ \mathbf{v}_{\max} &= Q^{-1}(\vec{\mu} - r_f\,\mathbf{1})\end{aligned}$$

It is also useful to have the association that the portfolio with weights

$$\mathbf{v}_P = a\,\mathbf{v}_{\min} + b\,\mathbf{v}_{aux} ,$$

has the following variance:

$$
\begin{aligned}
\sigma_P^2 &= \left(a\,\mathbf{v}_{\min} + b\,\mathbf{v}_{aux}\right)\cdot Q\left(a\,\mathbf{v}_{\min} + b\,\mathbf{v}_{aux}\right) = \\
&= a^2\,\mathbf{v}_{\min}\cdot Q\,\mathbf{v}_{\min} + b^2\,\mathbf{v}_{aux}\cdot Q\,\mathbf{v}_{aux} + 2ab\,\mathbf{v}_{\min}\cdot Q\,\mathbf{v}_{aux} = \\
&= a^2\,\mathbf{v}_{\min}\cdot Q\,\mathbf{v}_{\min} + b^2\,\mathbf{v}_{aux}\cdot Q\,\mathbf{v}_{aux} + 2ab\left(\sigma_{\min}^2 Q^{-1}\mathbf{1}QQ^{-1}\left(\vec{\mu} - \mu_{\min}\,\mathbf{1}\right)\right) = \\
&= a^2\,\mathbf{v}_{\min}\cdot Q\,\mathbf{v}_{\min} + b^2\,\mathbf{v}_{aux}\cdot Q\,\mathbf{v}_{aux} + 2ab\left(\sigma_{\min}^2\,\mathbf{1}Q^{-1}\left(\vec{\mu} - \mu_{\min}\,\mathbf{1}\right)\right) = \\
&= a^2\,\mathbf{v}_{\min}\cdot Q\,\mathbf{v}_{\min} + b^2\,\mathbf{v}_{aux}\cdot Q\,\mathbf{v}_{aux} + 2ab\sigma_{\min}^2\left(\mathbf{1}Q^{-1}\vec{\mu} - \mu_{\min}\,\mathbf{1}Q^{-1}\mathbf{1}\right) = \\
&= a^2\,\mathbf{v}_{\min}\cdot Q\,\mathbf{v}_{\min} + b^2\,\mathbf{v}_{aux}\cdot Q\,\mathbf{v}_{aux} + 2ab\sigma_{\min}^2\left(\frac{\mu_{\min}}{\sigma_{\min}^2} - \frac{\mu_{\min}}{\sigma_{\min}^2}\right) = \\
&= a^2\,\mathbf{v}_{\min}\cdot Q\,\mathbf{v}_{\min} + b^2\,\mathbf{v}_{aux}\cdot Q\,\mathbf{v}_{aux} = a^2\sigma_{\min}^2 + b^2\sigma_{aux}^2
\end{aligned}
$$

5 Markowitz portfolio

This portfolio is also known as the tangency portfolio and it is what we get if we make sure that the sum of the weights in $\mathbf{v}_{\max}$ is equal to 1. This is perfectly in line with our purpose. Since we are not allowed to keep more than 100% of our capital invested, we are forced to scale the weights so they sum to 1. We see that $\mathbf{v}_{\max}$ can be expressed as

$$\begin{aligned}
\mathbf{v}_{\max} &= Q^{-1}\left(\vec{\mu} - r_f\,\mathbf{1}\right) = \\
&= Q^{-1}\left(\vec{\mu} - \mu_{\min}\mathbf{1} + \mu_{\min}\mathbf{1} - r_f\,\mathbf{1}\right) = \\
&= Q^{-1}\left(\vec{\mu} - \mu_{\min}\mathbf{1}\right) + Q^{-1}\left(\mu_{\min}\mathbf{1} - r_f\,\mathbf{1}\right) = \\
&= \mathbf{v}_{aux} + \frac{\sigma_{\min}^2}{\sigma_{\min}^2} Q^{-1}\left(\mu_{\min} - r_f\right)\mathbf{1} = \\
&= \mathbf{v}_{aux} + \frac{\mu_{\min} - r_f}{\sigma_{\min}^2}\mathbf{v}_{\min} = \left[\varpi = \frac{\mu_{\min} - r_f}{\sigma_{\min}^2}\right] = \\
&= \mathbf{v}_{aux} + \varpi\,\mathbf{v}_{\min}
\end{aligned}$$

Remembering that the sum of weights of the auxiliary portfolio is 0, and the sum of weights of the minimum variance portfolio is 1, gives us the following expression for the Markowitz or the tangency portfolio:

$$\mathbf{v}_{Mz} = \frac{1}{\varpi}\mathbf{v}_{\max} = \mathbf{v}_{\min} + \frac{1}{\varpi}\mathbf{v}_{aux}$$

This results in the following expression for the expected return and variance for the Markowitz portfolio.

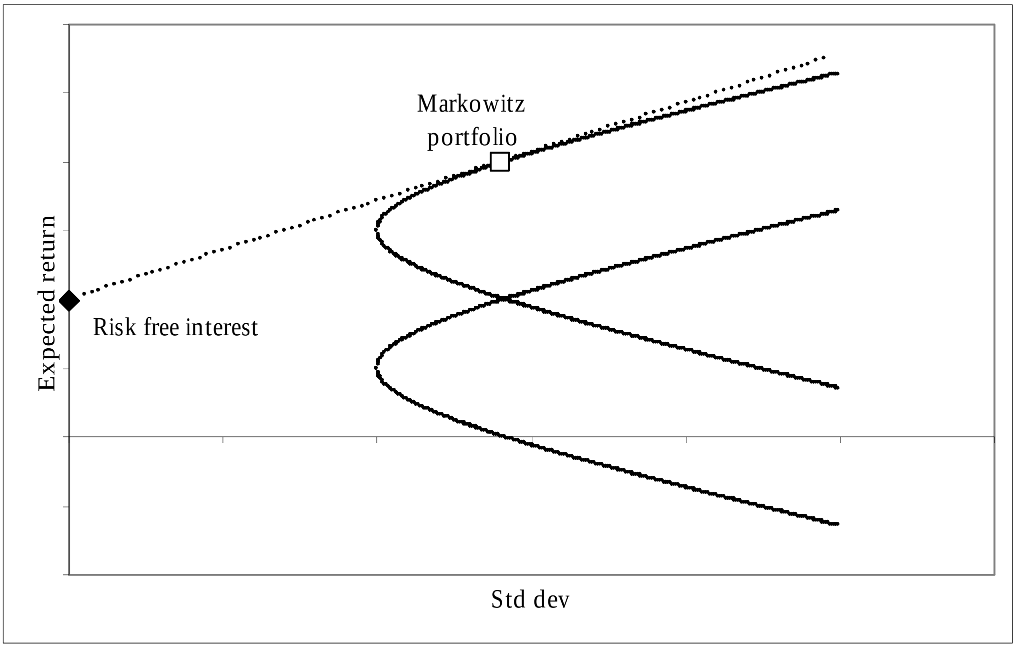
$$\begin{aligned}
\mu_{Mz} &= \vec{\mu}\cdot\mathbf{v}_{Mz} \\
\sigma_{Mz}^2 &= \mathbf{v}_{Mz}\cdot Q\,\mathbf{v}_{Mz}
\end{aligned}$$

5.1 Two different scenarios

As well as in the case with the minimum variance portfolio, we get two different scenarios even here. Remembering the expression of ϖ in the previous section:

$$\varpi = \frac{\mu_{\min} - r_f}{\sigma_{\min}^2}$$

This tells us that as long as $\mu_{\min} > r_f$, the weight reduction factor ϖ is positive, leading to positive weights in $\mathbf{v}_{\max}$. If, alternatively, the expected return of the minimum variance portfolio is less than the risk free interest, the concept of a tangency portfolio falls. The following illustration explains this.



This graph is intended to show the two different scenarios that may arise depending on the location of the effective front in relation to the risk free interest. We see that it is only in the case when the minimum variance portfolio (located at the leftmost position of the effective front) has a higher expected return than the risk free interest, that we are able to create a tangency portfolio.

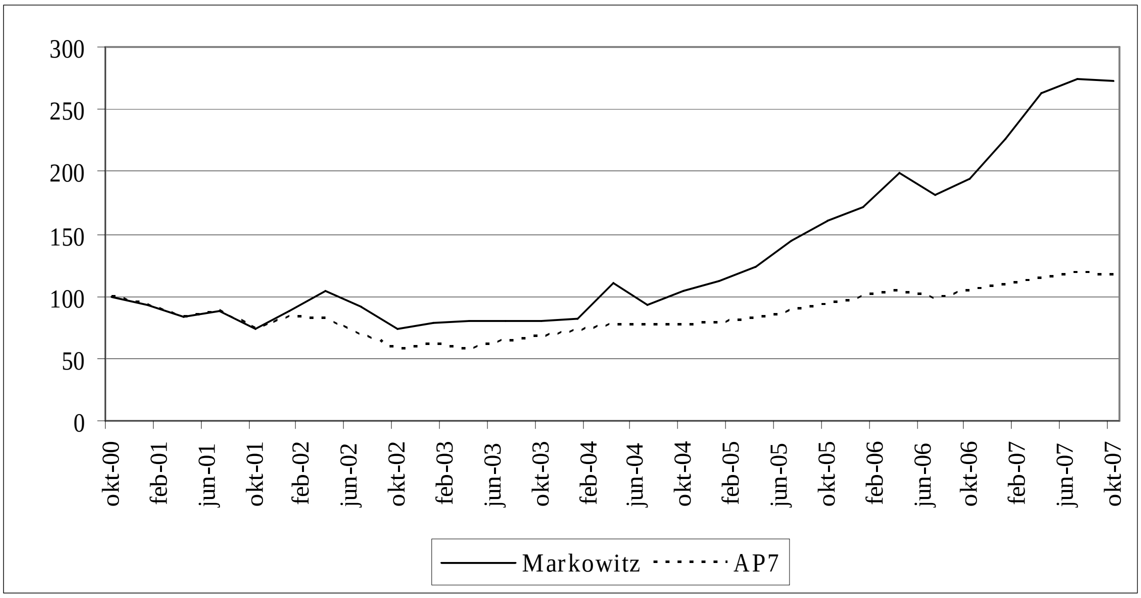
For the case when $\mu_{\min} \leq r_f$ we will choose to keep our money in cash at risk free interest.

5.2 Empirical results

The following table shows the resulting choices in how to invest our tangency/Markowitz portfolio for each calculated period:

Period	Risk free interest rate	Exp return from minimum variance portfolio	Resulting investment choice
2001 Q4	0,00076	0,00353	Markowitz portfolio
2002 Q1	0,00077	0,00634	Markowitz portfolio
2002 Q2	0,00074	0,00738	Markowitz portfolio
2002 Q3	0,00075	0,00498	Markowitz portfolio
2002 Q4	0,00076	0,00284	Markowitz portfolio
2003 Q1	0,00078	0,00115	Cash
2003 Q2	0,00076	0,00037	Cash
2003 Q3	0,00076	0,00073	Cash
2003 Q4	0,00073	0,00635	Markowitz portfolio
2004 Q1	0,00071	0,00843	Markowitz portfolio
2004 Q2	0,00069	0,00731	Markowitz portfolio
2004 Q3	0,00068	0,00354	Markowitz portfolio
2004 Q4	0,00061	0,00474	Markowitz portfolio
2005 Q1	0,00059	0,00396	Markowitz portfolio
2005 Q2	0,00057	0,00478	Markowitz portfolio
2005 Q3	0,00051	0,00577	Markowitz portfolio
2005 Q4	0,00048	0,00729	Markowitz portfolio
2006 Q1	0,00042	0,00730	Markowitz portfolio
2006 Q2	0,00039	0,00702	Markowitz portfolio
2006 Q3	0,00035	0,00618	Markowitz portfolio
2006 Q4	0,00035	0,00561	Markowitz portfolio
2007 Q1	0,00034	0,00771	Markowitz portfolio
2007 Q2	0,00034	0,00623	Markowitz portfolio
2007 Q3	0,00034	0,00673	Markowitz portfolio
2007 Q4	0,00037	0,00708	Markowitz portfolio

Following this strategy for our portfolio, would have given us a return of our investment of 172,5% in these seven years, 2000-10-01 to 2007-10-01. Remember that we kept our money invested at the default alternative the first year, 2000-10-01 to 2001-10-01, hence the start of our Markowitz strategy by Q4 2001. Depending on how we choose to calculate we get different results. Should we choose to start our calculation of our yearly yield using the Markowitz portfolio strategy, by calculating an increase from 100 SEK invested at 2000-10-01, with one year in the default choice, the Premium savings fund managed by AP7, and using our particular Markowitz approach during 2001-10-01 to 2007-10-01, we would receive a yearly yield of 15,4% in these seven years. We could also be so bold to say that we only would like to know the yearly yield for the time period when we were in charge, that is the six year period between 2001-10-01 and 2007-10-01. The default alternative had by the end of Q3 2001 reduced our 100 SEK to merely 73,22 SEK. If we now calculate our yearly yield from 73,22 to 272,5 SEK in only six years, it gives us the result of 24,5% per year. The resulting portfolios for each period will be found in Appendix A2.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Markowitz
AP7

6 Maximum drift with constraint on total stock weight

The amount of cash in a portfolio can be negative, which means that you borrow money to invest in other assets. The usual way to do this is to use the stocks in your portfolio as collateral for the loan. In the case of investing in funds, we have a theoretical possibility to take a loan to increase our investment, but in order to get the loan granted, we would have to put up with something else as collateral. In our specific case of investing in this particular part of the pension system, we do not even have the opportunity to increase our premium, since it is automatically paid once a year without our doing. What we can do, and should, is to search for the best way to make these premiums grow in accordance with our willingness to accept risk and our general belief of the development of the market in the forthcoming investment period. If we think that the stock market will have a strong progress or if we generally think that a higher risk is rewarded by a higher return of our investment, we would want to maximize the growth. If we assume that the total weight of our portfolio is h and we keep the remainder in cash, that is $-(h-1)$, our objective would be to maximize

$$\upsilon_P = r_f + (\vec{\mu} - r_f \mathbf{1})^T \mathbf{v} - \frac{1}{2}\mathbf{v}^T Q \mathbf{v}$$

under the constraint

$$\mathbf{1} \cdot \mathbf{v}_{\max h} = h.$$

Using Lagrange multipliers again we get the following expression

$$L(\mathbf{v}_{\max h}, \lambda) = r_f + (\vec{\mu} - r_f \mathbf{1})^T \mathbf{v}_{\max h} - \frac{1}{2}\mathbf{v}_{\max h}{}^T Q \mathbf{v}_{\max h} + \lambda (h - \mathbf{1} \cdot \mathbf{v}_{\max h}),$$

which leads to

$$\frac{\partial L}{\partial \mathbf{v}_{\max h}} = \vec{\mu} - r_f \mathbf{1} - Q \mathbf{v}_{\max h} - \lambda \mathbf{1} = 0, \text{ and } \mathbf{1} \cdot \mathbf{v}_{\max h} = h.$$

Rewriting the above expressions gives

$$
\begin{aligned}
\mathbf{1}\cdot\mathbf{v}_{\max h} &= \mathbf{1}\cdot Q^{-1}\left(\vec{\mu}-r_f\cdot\mathbf{1}-\lambda\cdot\mathbf{1}\right)=\left|Q^{-1}\left(\vec{\mu}-r_f\cdot\mathbf{1}\right)=\mathbf{v}_{\max}\right|=\\
&=\mathbf{1}\cdot\mathbf{v}_{\max}-\lambda\cdot\mathbf{1}\cdot Q^{-1}\mathbf{1}=\left\lceil\mathbf{1}\cdot\mathbf{v}_{\max}=\varpi\right\rceil=\\
&=\varpi-\lambda\cdot\mathbf{1}\cdot Q^{-1}\mathbf{1}=\left\lceil\mathbf{1}\cdot Q^{-1}\mathbf{1}=\frac{1}{\sigma^2_{\min}}\right\rceil=\\
&=\varpi-\frac{\lambda}{\sigma^2_{\min}}=h.
\end{aligned}
$$

Solving for λ gives

$$\lambda=\sigma^2_{\min}\left(\varpi-h\right)$$

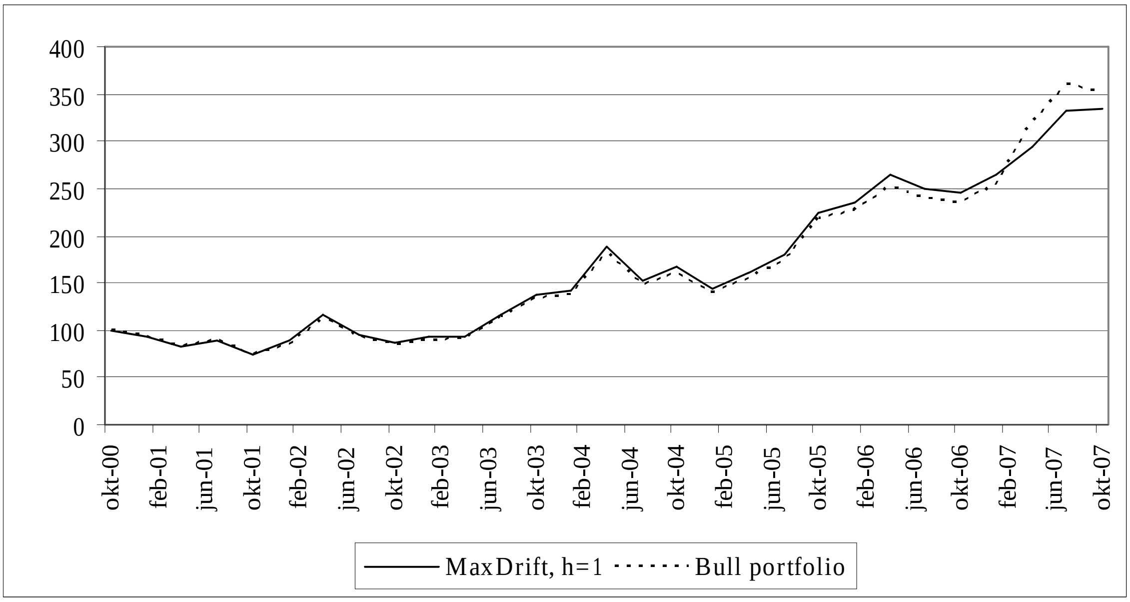
and inserting into the expression of $\mathbf{v}_{\max h}$ we get

$$
\begin{aligned}
\mathbf{v}_{\max h} &= Q^{-1}\left(\vec{\mu}-r_f\cdot\mathbf{1}-\sigma^2_{\min}\left(\varpi-h\right)\cdot\mathbf{1}\right)=\left\lceil\varpi=\frac{\mu_{\min}-r_f}{\sigma^2_{\min}}\right\rceil=\\
&=Q^{-1}\left(\vec{\mu}-r_f\cdot\mathbf{1}-\sigma^2_{\min}\left(\frac{\mu_{\min}-r_f}{\sigma^2_{\min}}-h\right)\cdot\mathbf{1}\right)=\\
&=Q^{-1}\left(\vec{\mu}-r_f\cdot\mathbf{1}-\left(\mu_{\min}-r_f-\sigma^2_{\min}h\right)\cdot\mathbf{1}\right)=\\
&=Q^{-1}\left(\vec{\mu}-r_f\cdot\mathbf{1}-\mu_{\min}\cdot\mathbf{1}+r_f\cdot\mathbf{1}+\sigma^2_{\min}h\cdot\mathbf{1}\right)=\\
&=Q^{-1}\left(\vec{\mu}-\mu_{\min}\cdot\mathbf{1}\right)+Q^{-1}\left(\sigma^2_{\min}h\cdot\mathbf{1}\right)=\\
&=Q^{-1}\left(\vec{\mu}-\mu_{\min}\cdot\mathbf{1}\right)+h\cdot\sigma^2_{\min}Q^{-1}\cdot\mathbf{1}=\\
&=\mathbf{v}_{aux}+h\cdot\mathbf{v}_{\min}.
\end{aligned}
$$

Our objective here was to maximize the drift under the constraint that the total weight of our portfolio was h and we kept the remainder in cash. If we really wanted to go for the highest possible returns we would choose $h=1$, that is, we always aim to keep the whole portfolio invested in stocks and nothing in cash. In our case this would mean that we intend to never let our portfolio contain any interest funds, only stock funds. Since we aren't able to sell short, $h=1$ is the most bullish approach we can take. The expressions for the expected return and variance of this portfolio are as follows.

$$
\begin{aligned}
\mu_{\max h} &= \vec{\mu}\cdot\mathbf{v}_{\max h}\\
\sigma^2_{\max h} &= \mathbf{v}_{\max h}\cdot Q\,\mathbf{v}_{\max h}
\end{aligned}
$$

Given that this is a portfolio for the very confident investor, it requires a very opportunistic opponent to match its expectations. We will construct a rather simple portfolio by selecting the one fund that had the highest return of all funds during the measurement period, as our investment choice for the coming period. This reference portfolio will be called the Bull portfolio. We will see that these two portfolios follow each other quite closely; in fact there are only four periods that these two strategies come up with a different set of funds. The resulting choices are available in full detail in Appendix A3. The result of these two investment strategies are displayed in the graph below.



We see that during this seven year period, both of these investments give a high return on our invested 100 SEK. Our calculated portfolio, Max Drift, h=1, gives a return of 232,8% in these years while the quite simply constructed portfolio gives a return of 251,5% for the same period. This corresponds to a yearly return of 18,7% and 19,7% for the Bull portfolio. Again, should we select only the return of the periods when we had our money invested in accordance with our strategies and not in the care of AP7, we get the following results: The Maximum drift portfolio with all stock funds, a return of 354,5% in six years (28,7% per year), and for the Bull portfolio, a return of 380% in six years (29,9% per year).

7 Constraint on total stock weight and portfolio volatility

By adding a constraint on the volatility of the portfolio one achieves the purpose of either adding a level of maximum risk or, as in the case we will show here, tracking the risk of an existing portfolio or fund, all with the intention of improving the return of our investment in proportion to the already existing one with the same risk. The fund we are about to create a replica of is naturally the default fund, the Premium savings fund, managed by AP7. This is accomplished by measuring the standard deviation of the Premium savings fund during our measurement period and maximize the drift, υ_P, of our portfolio under the constraint that our portfolio will have the same volatility as the Premium savings fund. We also would like to have a fully invested portfolio, as in the previous section with $h = 1$. Our objective will thus be to find the weights $\mathbf{v}_{AP7}$ that maximizes the drift under the constraints

$$\mathbf{1} \cdot \mathbf{v}_{AP7} = h \text{ and } \mathbf{v}_{AP7} \cdot Q \mathbf{v}_{AP7} = \sigma^2 .$$

We define

$$\mathbf{x} = Q^{1/2} \mathbf{v}_{AP7}$$
$$\mathbf{a} = Q^{-1/2} \left(\mu - r_f \mathbf{1} \right)$$
$$\mathbf{b} = Q^{-1/2} \mathbf{1}$$

Then our objective will be to maximize $\mathbf{a} \cdot \mathbf{x}$ subject to $|\mathbf{x}| = \sigma$, and $\mathbf{b} \cdot \mathbf{x} = h$.

Let

$$\mathbf{y} = \mathbf{x} - \frac{h}{|\mathbf{b}|^2} \mathbf{b} \text{, then}$$

$$\mathbf{y} \in \mathbf{b}^{\perp} = \{ \mathbf{z} \in \Re; \mathbf{b} \cdot \mathbf{z} = 0 \} \text{, and}$$

$$|\mathbf{y}|^2 = |\mathbf{x}|^2 - \frac{h^2}{|\mathbf{b}|^2} = \begin{bmatrix} |\mathbf{x}|^2 = \sigma^2 \\ \frac{1}{|\mathbf{b}|^2} = \frac{1}{\mathbf{1} \cdot Q^{-1} \mathbf{1}} = \sigma_{\min}^2 \end{bmatrix} = \sigma^2 - h^2 \sigma_{\min}^2 .$$

We have, by our definition of $\mathbf{y}$, that

$$\mathbf{a} \cdot \mathbf{x} = \mathbf{a} \cdot \mathbf{y} + h \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2} .$$

If we now let

$$\mathbf{d} = \mathbf{a} - \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\mathbf{b},$$

we have that $\mathbf{d} \in \mathbf{b}^{\perp}$ and

$$\begin{aligned}
\mathbf{d}\cdot\mathbf{y} &= \left(\mathbf{a} - \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\mathbf{b}\right)\left(\mathbf{x} - \frac{h}{|\mathbf{b}|^2}\mathbf{b}\right) = \\
&= \mathbf{a}\cdot\mathbf{x} - h\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2} - \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\mathbf{b}\cdot\mathbf{x} + \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}h = \\
&= \mathbf{a}\cdot\mathbf{x} - \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\mathbf{b}\cdot\mathbf{x} = \left\lceil \mathbf{b}\cdot\mathbf{x} = h \right\rceil = \\
&= \mathbf{a}\cdot\mathbf{x} - h\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2} = \mathbf{a}\cdot\mathbf{y}.
\end{aligned}$$

We shall therefore maximize $\mathbf{d}\cdot\mathbf{y}$ subject to $\mathbf{y} \in \mathbf{b}^{\perp}$ and our previous expression of $|\mathbf{y}|^2$, which leads to $\mathbf{y} = k\,\mathbf{d}$, where $k > 0$ and $k^2|\mathbf{d}|^2 = \sigma^2 - h^2\sigma_{\min}^2$.
In order to complete our calculations we could use the following relationship:

$$\begin{aligned}
\mathbf{a}\cdot\mathbf{b} &= \mathbf{1}\cdot Q^{-1}\left(\vec{\mu} - r_f\,\mathbf{1}\right) = \mathbf{1}\cdot Q^{-1}\vec{\mu} - r_f\,\mathbf{1}\cdot Q^{-1}\mathbf{1} = \\
&= \mathbf{1}\cdot Q^{-1}\vec{\mu} - \frac{r_f}{\sigma_{\min}^2} = \frac{\sigma_{\min}^2\,\mathbf{1}\cdot Q^{-1}\vec{\mu} - r_f}{\sigma_{\min}^2} = \\
&= \frac{\mu_{\min} - r_f}{\sigma_{\min}^2} = \varpi.
\end{aligned}$$

If we combine the following expressions, we get

$$\left.\begin{aligned}\mathbf{y} &= k\,\mathbf{d}\\ \mathbf{y} &= \mathbf{x} - \frac{h}{|\mathbf{b}|^2}\mathbf{b}\end{aligned}\right\} \Leftrightarrow \mathbf{x} = \frac{h}{|\mathbf{b}|^2}\mathbf{b} + k\,\mathbf{d} \Leftrightarrow \left\lceil \mathbf{x} = Q^{1/2}\mathbf{v}_{AP7} \right\rceil \Leftrightarrow$$

$$\begin{aligned}
Q^{1/2}\mathbf{v}_{AP7} &= \frac{h}{|\mathbf{b}|^2}\mathbf{b} + k\left(\mathbf{a} - \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\mathbf{b}\right) = \\
&= h\frac{Q^{-1/2}\mathbf{1}}{|\mathbf{b}|^2} + k\left(\mathbf{a} - \frac{\varpi}{|\mathbf{b}|^2}\mathbf{b}\right) \Leftrightarrow \\
\Leftrightarrow\ \mathbf{v}_{AP7} &= h\frac{Q^{-1}\mathbf{1}}{|\mathbf{b}|^2} + k\,Q^{-1/2}\left(Q^{-1/2}\left(\vec{\mu} - r_f\,\mathbf{1}\right) - \frac{\varpi}{|\mathbf{b}|^2}Q^{-1/2}\mathbf{1}\right) = \\
&= h\sigma^2_{\min}Q^{-1}\mathbf{1} + k\left(Q^{-1}\left(\vec{\mu} - r_f\,\mathbf{1}\right) - \left(\mu_{\min} - r_f\right)Q^{-1}\mathbf{1}\right) = \\
&= h\,\mathbf{v}_{\min} + k\left(Q^{-1}\vec{\mu} - r_f Q^{-1}\mathbf{1} - Q^{-1}\mu_{\min}\mathbf{1} + r_f Q^{-1}\mathbf{1}\right) = \\
&= h\,\mathbf{v}_{\min} + k\,Q^{-1}\left(\vec{\mu} - \mu_{\min}\mathbf{1}\right) = \\
&= h\,\mathbf{v}_{\min} + k\,\mathbf{v}_{aux}
\end{aligned}$$

In order to get a proper expression for k, we first need to calculate $|\mathbf{d}|^2$. By our definition of $\mathbf{d}$, we get that

$$\begin{aligned}
|\mathbf{d}|^2 &= |\mathbf{a}|^2 - \frac{(\mathbf{a}\cdot\mathbf{b})^2}{|\mathbf{b}|^2} = \sigma^2_{\max} - \varpi^2\sigma^2_{\min} \Leftrightarrow \\
\sigma^2_{\max} &= |\mathbf{d}|^2 + \varpi^2\sigma^2_{\min}
\end{aligned}$$

and recalling that the portfolio with weights $\mathbf{v}_{\max} = \varpi\,\mathbf{v}_{\min} + \mathbf{v}_{aux}$ has variance

$$\sigma^2_{\max} = \varpi^2\sigma^2_{\min} + \sigma^2_{aux},$$

gives us that $|\mathbf{d}|^2 = \sigma^2_{aux}$.

This inserted in to $k^2|\mathbf{d}|^2 = \sigma^2 - h^2\sigma^2_{\min}$ gives us the following expression for k.

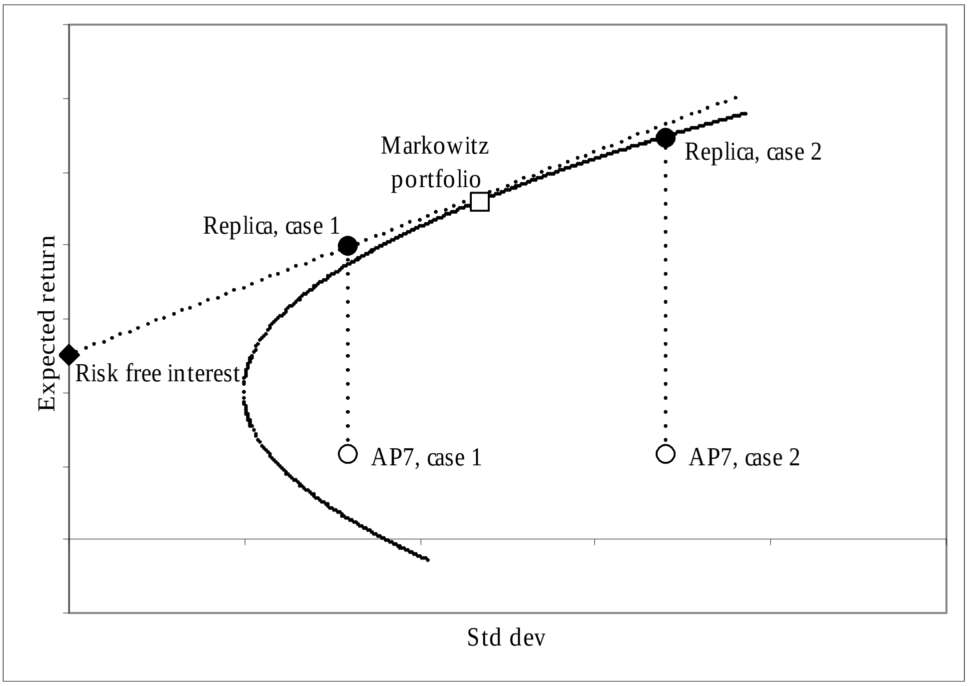
$$\begin{aligned}
k^2|\mathbf{d}|^2 &= \sigma^2 - h^2\sigma^2_{\min} \Leftrightarrow \\
k^2 &= \frac{\sigma^2 - h^2\sigma^2_{\min}}{\sigma^2_{aux}} \Rightarrow \\
k &= \frac{\sqrt{\sigma^2 - h^2\sigma^2_{\min}}}{\sigma_{aux}}
\end{aligned}$$

Now we have a valid expression for the weights of the portfolio that tracks a given variance:

$$\mathbf{v}_{AP7} = h\mathbf{v}_{\min} + \frac{\sqrt{\sigma^2 - h^2\sigma_{\min}^2}}{\sigma_{aux}}\mathbf{v}_{aux}.$$

7.1 Two different scenarios

Even this case has the possibility of two different portfolios, depending on the variance we are tracking. If the variance of our replicating portfolio is smaller than the variance of the Markowitz portfolio, it gives us better expected return to invest only a part k in the Markowitz portfolio and to keep $(1-k)$ in cash, as in case 1 in the figure below. If the variance of the Premium savings fund is larger than the Markowitz portfolio, we use the above calculations

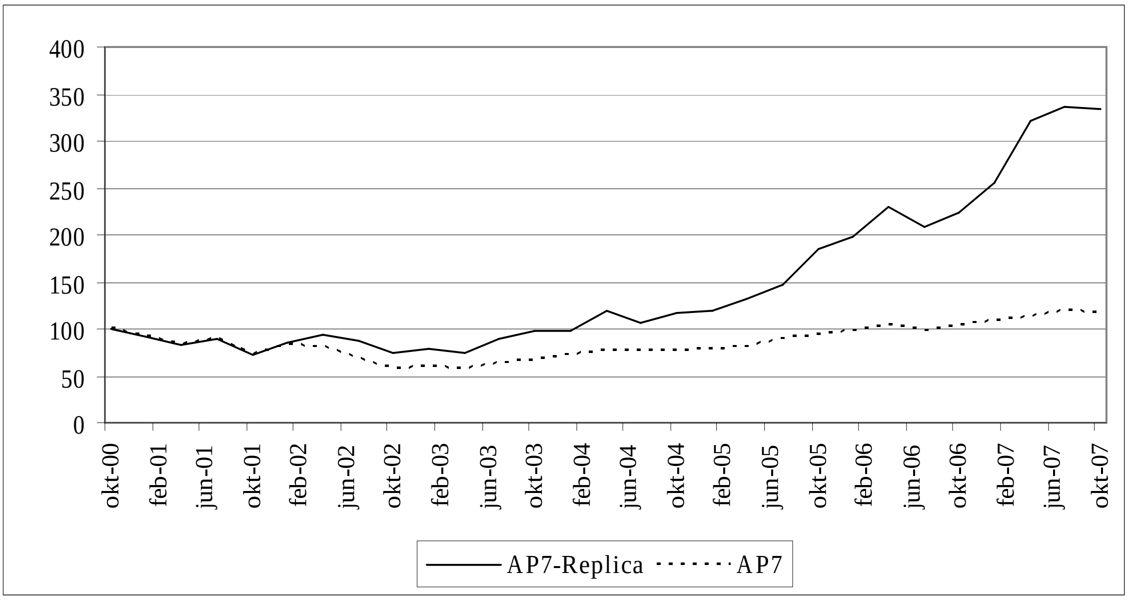


7.2 Empirical results

This table shows the standard deviation calculated for the Premium savings fund for each period and the corresponding standard deviation of the Markowitz portfolio. If the standard deviation of the fund managed by AP7 is lower than the Markowitz portfolio, the proper investment decision should be to use a linear combination of the Markowitz portfolio and the risk free interest instead of the AP7-replica, derived from the above calculations. The resulting portfolios using this strategy are available in full detail in appendix A4.

Period	Std dev of the original AP7	Std dev of Markowitz portfolio	Resulting investment choice
2001 Q4	0,0219	0,0234	Linear combination
2002 Q1	0,0223	0,0309	Linear combination
2002 Q2	0,0220	0,0306	Linear combination
2002 Q3	0,0220	0,0299	Linear combination
2002 Q4	0,0230	0,0287	Linear combination
2003 Q1	0,0238	0,0003	AP7-replica
2003 Q2	0,0242	0,0003	AP7-replica
2003 Q3	0,0243	0,0003	AP7-replica
2003 Q4	0,0236	0,0422	Linear combination
2004 Q1	0,0231	0,0398	Linear combination
2004 Q2	0,0231	0,0324	Linear combination
2004 Q3	0,0227	0,0209	AP7-replica
2004 Q4	0,0215	0,0170	AP7-replica
2005 Q1	0,0212	0,0145	AP7-replica
2005 Q2	0,0205	0,0150	AP7-replica
2005 Q3	0,0194	0,0142	AP7-replica
2005 Q4	0,0174	0,0132	AP7-replica
2006 Q1	0,0158	0,0139	AP7-replica
2006 Q2	0,0134	0,0100	AP7-replica
2006 Q3	0,0136	0,0137	Linear combination
2006 Q4	0,0137	0,0121	AP7-replica
2007 Q1	0,0136	0,0107	AP7-replica
2007 Q2	0,0138	0,0150	Linear combination
2007 Q3	0,0137	0,0157	Linear combination
2007 Q4	0,0140	0,0175	Linear combination

The graph below shows the results of the investment following the previous table. We see that while the original fund barely manages to show profit for the first six years, our portfolio shows a remarkable increase, especially during the last few years. The original fund presents a growth from 100 SEK to 118 SEK during this seven-year period resulting in a annul return of 2,4%. Using the above calculations we manage to boost our initial 100 SEK to 332,8 SEK, resulting in a annul return of 18,7% during the same period. Keep in mind that our portfolio invested all of its assets in the Premium savings fund for the first year.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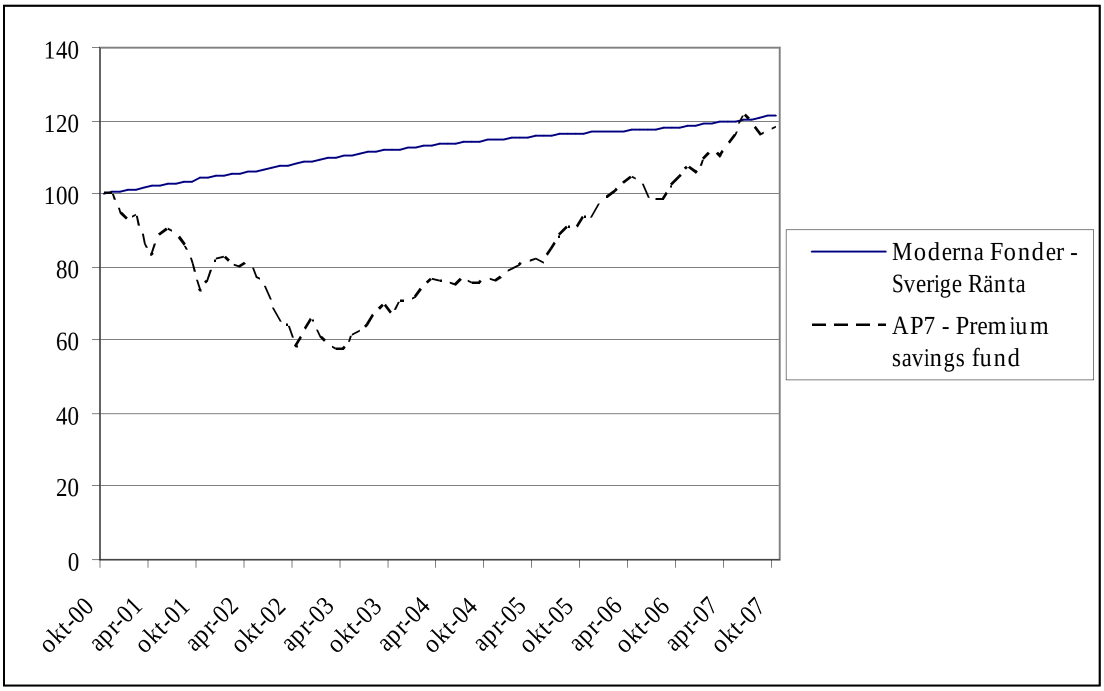
AP7-Replica
AP7

8 Conclusion

There certainly seems to be an economic incentive to pay a little attention to the selection of funds, instead of trusting the default choice. If we summarize all the portfolios that we have created and check their return in comparison to the Premium savings fund, we see that from the time of separation (the time when we decided that we had enough data to start to compute our portfolios, instead of keeping our investment in the care of AP7 – in our case we spent the first year gathering data.), all our portfolios by far, outperform the Premium savings fund, as seen in the first table below. That is with the exception of our risk free interest fund. But even with a very safe bet, as the selection of an interest fund, the Premium savings fund doesn't do that much better, seen from the very start of the premium pension system. If we select the one interest fund that has dominated our substitute for the risk free interest rate (*Moderna Fonder – Sverige Ränta*) and compare it to the Premium savings fund and their progress from day one we see that the development of 100 SEK is not that different in return after seven years, although the path is very different. One could of course argue that we are using hindsight when we choose the same fund that we used to replace the risk free interest in our calculations, but the reason that it was chosen was that its variance was the lowest - we took no consideration to the return. Since all interest funds should behave quite similar, we may keep this one just for illustrational purposes.

Portfolios	Total return Oct 2001 - Oct 2007	Average return, Annually	Std. dev. of total return
Risk Free Interest	15,9%	2,5%	0,0012
Minimum variance	160,8%	17,3%	0,0358
Markowitz	272,2%	24,5%	0,0502
AP7-Replica	354,5%	28,7%	0,0469
Maximum drift	354,5%	28,7%	0,0635
Bull	380,0%	29,9%	0,0652
AP7	61,1%	8,3%	0,0348

The table shows the total return of each portfolio during the six years that we calculated and the corresponding annual return since the portfolios were separated by October 2001.



Graph showing the development of 100 SEK in the case of an interest rate fund and the Premium savings fund seen from the start of the fund based premium savings system.

Below is a table showing the different annual return our portfolios yielded and since they all were invested in the Premium savings fund the first year, they all share the same return for that period.

Annual return From To	Oct 2000 Oct 2001	Oct 2001 Oct 2002	Oct 2002 Oct 2003	Oct 2003 Oct 2004	Oct 2004 Oct 2005	Oct 2005 Oct 2006	Oct 2006 Oct 2007
Risk Free Interest	-26,8%	3,2%	3,6%	2,3%	1,6%	1,6%	2,7%
Minimum variance	-26,8%	-10,2%	14,1%	30,2%	47,9%	18,8%	11,3%
Markowitz	-26,8%	1,0%	9,1%	28,9%	54,8%	20,2%	40,9%
AP7-Replica	-26,8%	2,0%	30,5%	20,3%	57,6%	20,6%	49,3%
Maximum drift	-26,8%	18,9%	58,4%	20,6%	34,9%	9,4%	35,6%
Bull	-26,8%	15,4%	58,4%	20,6%	34,9%	7,6%	50,1%
AP7	-26,8%	-21,0%	15,9%	13,9%	22,5%	11,6%	13,0%

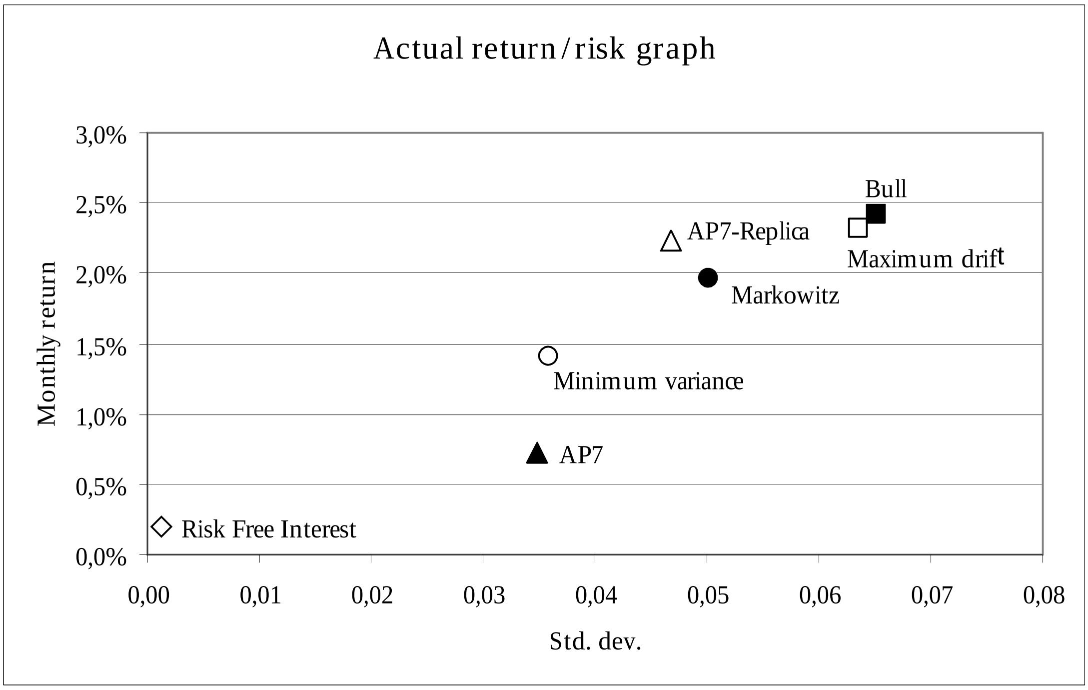
This table shows the annual return for each portfolio, including the default choice. Since all portfolios were untouched the first year they all show the same decline as the Premium savings fund by AP7.

It might also be of interest to see how the actual return/risk diagram became. The following graph shows the monthly return and its standard deviation and we have some interesting facts to reflect on.

- First of all we see that we failed in our attempt to imitate the variance of the default fund. Our AP7-replica portfolio has a higher variance that the original Premium savings fund.
- We might also notice that our Markowitz portfolio, which was designed to maximize the Sharpe ratio, is clearly beaten in that attempt by our AP7-replica portfolio.
- Unfortunately, the portfolio without any mathematics involved (the Bull portfolio), has a higher return than our Maximum drift portfolio. This might be possible due to the fact that this single fund is a portfolio in its own since that is the definition of a fund. This

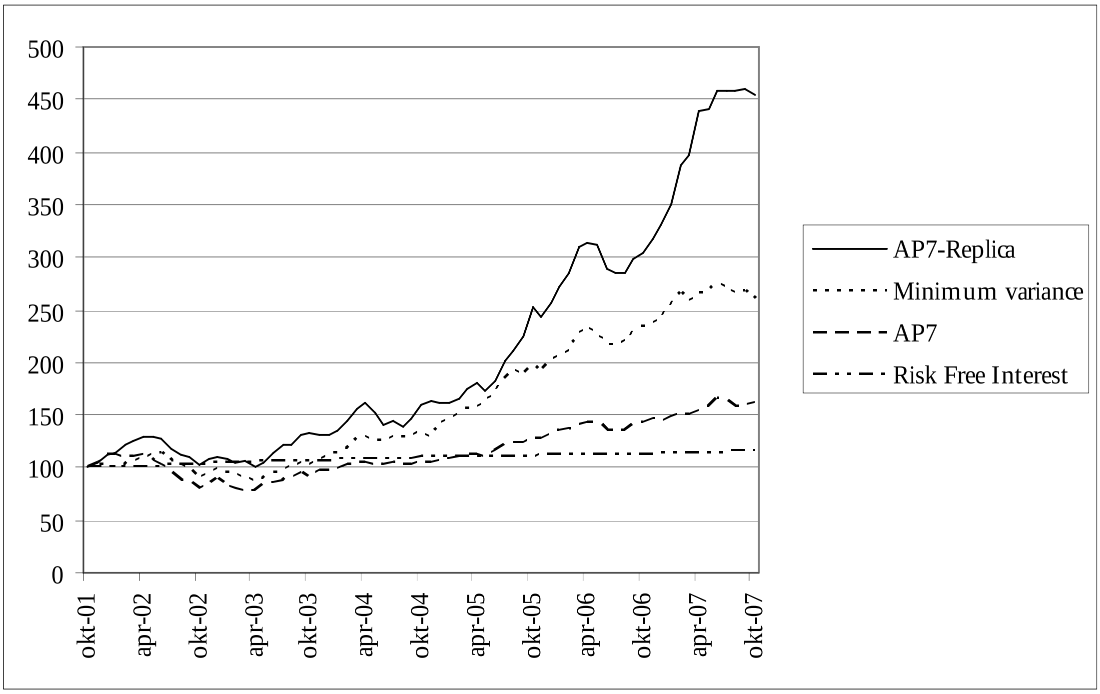
would probably not be the case if we would do the same calculations using single stocks instead of funds. We can only hope that the slightly higher risk eventually will take its toll.

- After that setback it is quite appealing to see that our Minimum variance portfolio is such a good alternative to the default choice, and with such a close match in risk as well.

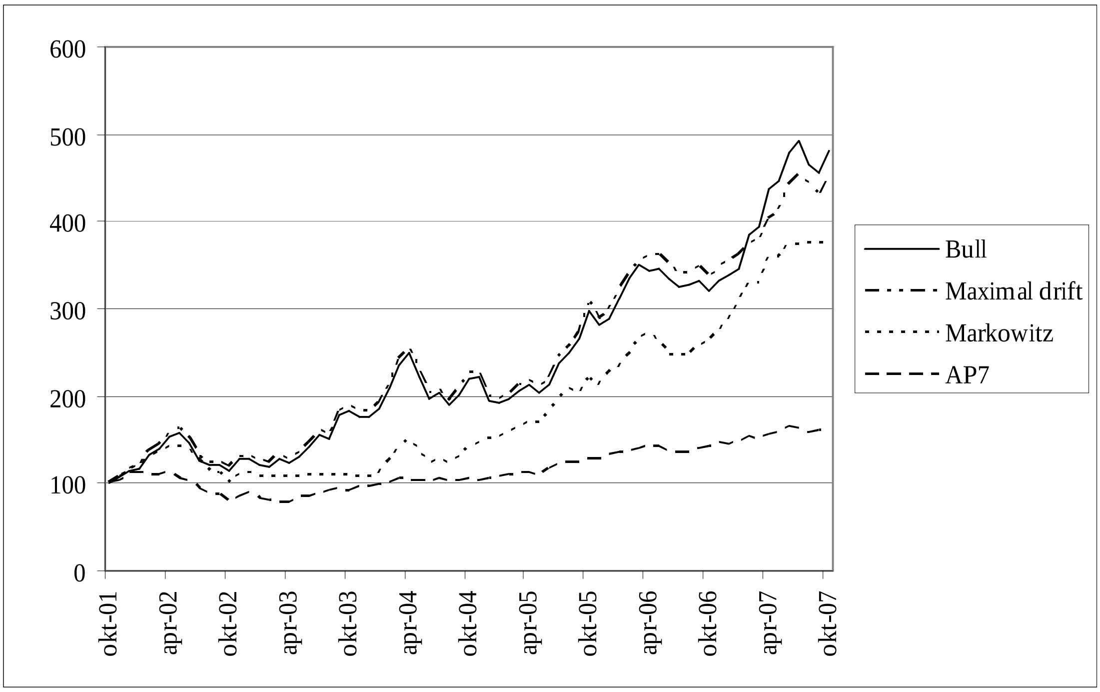


The graph shows the actual monthly return/standard deviation of returns for the calculated portfolios since the portfolios were separated by October 2001

Finally we present the development of all our portfolios, and we start at the time when we first created our own portfolios, at October 2001.



This shows the development of 100 SEK from the time of separation of the portfolios.



This shows the development of 100 SEK from the time of separation of the portfolios.

9 Appendix A1

The minimum variance portfolio of each period and the corresponding risk free interest.

2001 Q4	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00114	0,0138	0,72
Morgan Stanley SICAV European Equity Fund	-0,00107	0,0160	0,03
SGAM Fund Equities Switzerland	-0,00295	0,0188	0,08
Danske Fonder Global Index	-0,00375	0,0199	0,07
Carnegie Fund - WorldWide Sub-Fund	-0,00730	0,0224	0,10
Minimum variance portfolio	-0,00044	0,01288	1,00
Risk free interest			
Danske Fonder - Sverige Likviditet	0,00076		

2002 Q1	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00107	0,0136	0,65
Robur Realinvest	0,00027	0,0164	0,13
SGAM Fund Equities Switzerland	-0,00134	0,0190	0,08
Pictet Funds(LUX) - Water	0,00248	0,0191	0,06
Aktia Capital	0,00108	0,0221	0,08
Minimum variance portfolio	0,00086	0,01246	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00077		

2002 Q2	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00172	0,0134	0,59
Robur Realinvest	0,00103	0,0157	0,17
Pictet Funds(LUX) - Water	0,00113	0,0191	0,08
Aktia Capital	0,00180	0,0215	0,12
Carnegie Fund - WorldWide Sub-Fund	-0,00437	0,0222	0,04
Minimum variance portfolio	0,00132	0,0121	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00074		

2002 Q3	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00172	0,0147	0,55
Robur Realinvest	0,00044	0,0163	0,19
Pictet Funds(LUX) - Water	-0,00030	0,0195	0,08
Aktia Capital	0,00062	0,0209	0,17
Carnegie Fund - WorldWide Sub-Fund	-0,00529	0,0222	0,01
Minimum variance portfolio	0,00106	0,0133	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00075		

2002 Q4	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00018	0,0164	0,49
Robur Realinvest	-0,00106	0,0169	0,25
Aktia Capital	-0,00065	0,0213	0,19
Carnegie Fund - WorldWide Sub-Fund	-0,00600	0,0223	0,07
Minimum variance portfolio	-0,00072	0,0146	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00076		

2003 Q1	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00069	0,0162	0,54
Robur Realinvest	-0,00047	0,0174	0,21
Aktia Capital	0,00013	0,0219	0,16
Pictet Funds(LUX) - Water	-0,00210	0,0219	0,05
Carnegie Fund - WorldWide Sub-Fund	-0,00542	0,0226	0,04
Minimum variance portfolio	-0,00002	0,0148	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00078		

2003 Q2	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	-0,00024	0,0163	0,51
Robur Realinvest	-0,00087	0,0172	0,25
Aktia Capital	-0,00034	0,0216	0,17
Pictet Funds(LUX) - Water	-0,00264	0,0228	0,07
Minimum variance portfolio	-0,00058	0,0148	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00076		

2003 Q3	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00071	0,0164	0,48
Robur Realinvest	-0,00039	0,0169	0,28
Aktia Capital	0,00031	0,0211	0,17
Pictet Funds(LUX) - Water	-0,00178	0,0229	0,07
Minimum variance portfolio	0,00016	0,0145	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00076		

2003 Q4	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00078	0,0160	0,50
Robur Realinvest	0,00026	0,0168	0,27
Aktia Capital	0,00089	0,0209	0,17
Pictet Funds(LUX) - Water	-0,00165	0,0227	0,06
Minimum variance portfolio	0,00052	0,0144	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00073		

2004 Q1	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00124	0,0161	0,53
Robur Realinvest	0,00072	0,0171	0,22
Aktia Capital	0,00157	0,0203	0,18
Pictet Funds(LUX) - Water	-0,00196	0,0225	0,05
HQ Strategifond	-0,00086	0,0245	0,02
Minimum variance portfolio	0,00098	0,0146	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00071		

2004 Q2	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00161	0,0162	0,51
Robur Realinvest	0,00136	0,0168	0,23
Aktia Capital	0,00255	0,0202	0,14
HQ Strategifond	0,00039	0,0237	0,01
Seligson & Co Global Top 25 Pharmaceuticals	-0,00212	0,0239	0,11
Minimum variance portfolio	0,00126	0,0146	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00069		

2004 Q3	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00144	0,0165	0,33
Länsförsäkringar Fastighetsfond	0,00223	0,0174	0,33
Aktia Capital	0,00203	0,0187	0,10
Morgan Stanley SICAV Global Brands Fund	-0,00009	0,0202	0,18
Seligson & Co Global Top 25 Pharmaceuticals	-0,00223	0,0235	0,06
Minimum variance portfolio	0,00127	0,0136	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00068		

2004 Q4	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00231	0,0160	0,32
Länsförsäkringar Fastighetsfond	0,00399	0,0166	0,37
Mir Quality Growth SICAV - Europe Quality Growth Fund	-0,00096	0,0179	0,05
AXA Rosenberg Pacific Ex-Japan Equity Alpha Fund	0,00134	0,0200	0,08
Morgan Stanley SICAV Global Brands Fund	-0,00013	0,0203	0,18
Minimum variance portfolio	0,00225	0,0129	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00061		

2005 Q1	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00298	0,0160	0,30
Länsförsäkringar Fastighetsfond	0,00427	0,0164	0,38
Mir Quality Growth SICAV - Europe Quality Growth Fund	-0,00092	0,0176	0,05
AXA Rosenberg Pacific Ex-Japan Equity Alpha Fund	0,00069	0,0195	0,09
Morgan Stanley SICAV Global Brands Fund	-0,00013	0,0201	0,18
Minimum variance portfolio	0,00251	0,0129	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00059		

2005 Q2	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00252	0,0163	0,30
Länsförsäkringar Fastighetsfond	0,00425	0,0167	0,38
AXA Rosenberg Pacific Ex-Japan Equity Alpha Fund	0,00107	0,0197	0,06
Morgan Stanley SICAV Global Brands Fund	-0,00035	0,0200	0,17
East Capital Östeuropafonden	0,00538	0,0208	0,09
Minimum variance portfolio	0,00286	0,0130	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00059		

2005 Q3	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00350	0,0158	0,24
ODIN Finland	0,00421	0,0165	0,18
Länsförsäkringar Fastighetsfond	0,00551	0,0169	0,26
Morgan Stanley SICAV Global Brands Fund	0,00080	0,0192	0,20
East Capital Östeuropafonden	0,00617	0,0194	0,12
Minimum variance portfolio	0,00393	0,0123	1,00
Risk free interest			
Handelsbanken Fonder AB - Lux Ränta	0,00051		

2005 Q4	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00470	0,0140	0,41
ODIN Finland	0,00612	0,0156	0,16
Länsförsäkringar Fastighetsfond	0,00696	0,0169	0,22
Morgan Stanley SICAV Global Brands Fund	0,00176	0,0171	0,10
Seligson & Co Global Top 25 Pharmaceuticals	0,00142	0,0190	0,11
Minimum variance portfolio	0,00477	0,0113	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00048		

2006 Q1	Exp. weekly return	Std. dev.	Weight
Morgan Stanley SICAV European Property Fund	0,00442	0,0147	0,37
BL - Equities Europe	0,00308	0,0149	0,15
ODIN Finland	0,00571	0,0152	0,15
Länsförsäkringar Fastighetsfond	0,00622	0,0166	0,19
Seligson & Co Global Top 25 Pharmaceuticals	0,00166	0,0184	0,14
Minimum variance portfolio	0,00437	0,0115	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00042		

2006 Q2	Exp. weekly return	Std. dev.	Weight
BL - Equities Europe	0,00421	0,0123	0,16
Morgan Stanley SICAV Global Brands Fund	0,00287	0,0132	0,14
Catella Trygghetsfond	0,00478	0,0141	0,24
Morgan Stanley SICAV European Property Fund	0,00627	0,0144	0,17
East Capital Baltikumfonden	0,00645	0,0154	0,29
Minimum variance portfolio	0,00516	0,0942	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00039		

2006 Q3	Exp. weekly return	Std. dev.	Weight
BL - Equities Europe	0,00327	0,0132	0,23
Morgan Stanley SICAV Global Brands Fund	0,00206	0,0134	0,20
FIM Rento Fritid Placeringsfond	0,00311	0,0144	0,09
Öhman Medicafond	0,00142	0,0153	0,20
East Capital Baltikumfonden	0,00482	0,0156	0,28
Minimum variance portfolio	0,00308	0,0109	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00035		

2006 Q4	Exp. weekly return	Std. dev.	Weight
BL - Equities Europe	0,00361	0,0128	0,31
European Quality Fund SICAV - European Equity Fund	0,00344	0,0137	0,08
Morgan Stanley SICAV Global Brands Fund	0,00248	0,0137	0,06
East Capital Baltikumfonden	0,00476	0,0139	0,39
Öhman Medicafond	0,00210	0,0151	0,16
Minimum variance portfolio	0,00373	0,0102	1,00
Risk free interest			
Handelsbanken Fonder AB - Lux Ränta	0,00035		

2007 Q1	Exp. weekly return	Std. dev.	Weight
Linde Partners Value Fund Global	0,00236	0,0113	0,21
Linde Partners Value Fund Blue Chip Value	0,00220	0,0115	0,14
BL - Equities Europe	0,00338	0,0126	0,18
East Capital Baltikumfonden	0,00532	0,0138	0,36
Öhman Medicafond	0,00173	0,0153	0,11
Minimum variance portfolio	0,00352	0,0099	1,00
Risk free interest			
Handelsbanken Fonder AB - Lux Ränta	0,00034		

2007 Q2	Exp. weekly return	Std. dev.	Weight
Linde Partners Value Fund Global	0,00221	0,0114	0,39
Linde Partners Value Fund Blue Chip Value	0,00195	0,0115	0,20
European Quality Fund SICAV - European Equity Fund	0,00332	0,0130	0,09
Öhman Medicafond	0,00162	0,0151	0,10
East Capital Baltikumfonden	0,00427	0,0163	0,22
Minimum variance portfolio	0,00266	0,0106	1,00
Risk free interest			
Handelsbanken Fonder AB - Lux Ränta	0,00034		

2007 Q3	Exp. weekly return	Std. dev.	Weight
Linde Partners Value Fund Global	0,00228	0,0123	0,43
Linde Partners Value Fund Blue Chip Value	0,00197	0,0125	0,09
Morgan Stanley SICAV Global Brands Fund	0,00214	0,0142	0,11
Öhman Medicafond	0,00153	0,0152	0,13
East Capital Baltikumfonden	0,00499	0,0166	0,24
Minimum variance portfolio	0,00279	0,0114	1,00
Risk free interest			
Moderna Fonder - Sverige Ränta	0,00034		

2007 Q4	Exp. weekly return	Std. dev.	Weight
Linde Partners Value Fund Global	0,00200	0,0126	0,53
Linde Partners Value Fund Blue Chip Value	0,00226	0,0128	0,11
Morgan Stanley SICAV Global Brands Fund	0,00213	0,0142	0,12
Öhman Medicafond	0,00119	0,0150	0,14
Länsförsäkringar Nordamerikafond	0,00066	0,0161	0,10
Minimum variance portfolio	0,00180	0,0180	1,00
Risk free interest			
Handelsbanken Fonder AB - Lux Ränta	0,00037		

10 Appendix A2

The Markowitz portfolio of each period and in the case of a non existing Markowitz portfolio, we use the corresponding risk free interest for that period.

2001 Q4	Exp. weekly return	Std. dev.	Weight
Robur Skogsfond	0,00355	0,0331	0,41
Pictet Funds(LUX) - Water	0,00242	0,0206	0,51
HQ Rysslandsfond	0,00343	0,0545	0,08
Markowitz portfolio	0,00296	0,0234	1,00

2002 Q1	Exp. weekly return	Std. dev.	Weight
Robur Skogsfond	0,00549	0,0311	0,62
HQ Rysslandsfond	0,00773	0,0513	0,38
Markowitz portfolio	0,00634	0,0309	1,00

2002 Q2	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00963	0,0490	0,44
Robur Skogsfond	0,00560	0,0305	0,56
Markowitz portfolio	0,00738	0,0306	1,00

2002 Q3	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00643	0,0492	0,39
Robur Skogsfond	0,00406	0,0295	0,61
Markowitz portfolio	0,00498	0,0299	1,00

2002 Q4	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00501	0,0469	0,55
Morgan Stanley SICAV European Property Fund	0,00018	0,0164	0,45
Markowitz portfolio	0,00284	0,0287	1,00

2003 Q1	Exp. weekly return	Std. dev.	Weight
Moderna Fonder - Sverige Ränta	0,00078	0,0003	1,00
Markowitz portfolio	0,00078	0,0003	1,00

2003 Q2	Exp. weekly return	Std. dev.	Weight
Moderna Fonder - Sverige Ränta	0,00076	0,0003	1,00
Markowitz portfolio	0,00076	0,0003	1,00

2003 Q3	Exp. weekly return	Std. dev.	Weight
Moderna Fonder - Sverige Ränta	0,00076	0,0003	1,00
Markowitz portfolio	0,00076	0,0003	1,00

2003 Q4	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00635	0,0422	1,00
Markowitz portfolio	0,00635	0,0422	1,00

2004 Q1	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00843	0,0398	1,00
Markowitz portfolio	0,00843	0,0398	1,00

2004 Q2	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00842	0,0381	0,81
Aktia Capital	0,00255	0,0202	0,19
Markowitz portfolio	0,00731	0,0324	1,00

2004 Q3	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00534	0,0385	0,42
Länsförsäkringar Fastighetsfond	0,00223	0,0174	0,58
Markowitz portfolio	0,00354	0,0209	1,00

2004 Q4	Exp. weekly return	Std. dev.	Weight
Länsförsäkringar Fastighetsfond	0,00399	0,0166	0,74
HQ Rysslandsfond	0,00688	0,0367	0,26
Markowitz portfolio	0,00474	0,0170	1,00

2005 Q1	Exp. weekly return	Std. dev.	Weight
Länsförsäkringar Fastighetsfond	0,00427	0,0164	0,75
Morgan Stanley SICAV European Property Fund	0,00298	0,0160	0,20
HQ Rysslandsfond	0,00355	0,0372	0,03
AXA Rosenberg Japan Small Cap Alpha Fund	0,00272	0,0302	0,02
Markowitz portfolio	0,00396	0,0145	1,00

2005 Q2	Exp. weekly return	Std. dev.	Weight
East Capital Östeuropafonden	0,00538	0,0208	0,47
Länsförsäkringar Fastighetsfond	0,00425	0,0167	0,53
Markowitz portfolio	0,00478	0,0150	1,00

2005 Q3	Exp. weekly return	Std. dev.	Weight
Länsförsäkringar Fastighetsfond	0,00551	0,0169	0,52
East Capital Östeuropafonden	0,00617	0,0194	0,45
ODIN Finland	0,00421	0,0165	0,03
Markowitz portfolio	0,00577	0,0142	1,00

2005 Q4	Exp. weekly return	Std. dev.	Weight
Länsförsäkringar Fastighetsfond	0,00696	0,0169	0,38
East Capital Östeuropafonden	0,00805	0,0198	0,29
ODIN Norge	0,00902	0,0231	0,13
ODIN Finland	0,00612	0,0156	0,14
Morgan Stanley SICAV European Property Fund	0,00470	0,0140	0,06
Markowitz portfolio	0,00729	0,0132	1,00

2006 Q1	Exp. weekly return	Std. dev.	Weight
East Capital Östeuropafonden	0,00868	0,0205	0,35
ODIN Norge	0,00883	0,0232	0,12
Länsförsäkringar Fastighetsfond	0,00622	0,0166	0,31
ODIN Finland	0,00571	0,0152	0,07
ODIN Sverige	0,00585	0,0158	0,15
Markowitz portfolio	0,00730	0,0139	1,00

2006 Q2	Exp. weekly return	Std. dev.	Weight
ODIN Sverige	0,00730	0,0150	0,18
Länsförsäkringar Fastighetsfond	0,00782	0,0168	0,26
Morgan Stanley SICAV European Property Fund	0,00627	0,0144	0,11
East Capital Baltikumfonden	0,00645	0,0154	0,35
Spiltan Aktiefond Sverige	0,00730	0,0179	0,10
Markowitz portfolio	0,00702	0,0100	1,00

2006 Q3	Exp. weekly return	Std. dev.	Weight
Fondita Nordic Small Cap Placeringsfond	0,00694	0,0193	0,19
ODIN Norge	0,00860	0,0245	0,16
Länsförsäkringar Fastighetsfond	0,00640	0,0205	0,22
East Capital Baltikumfonden	0,00482	0,0156	0,43
Markowitz portfolio	0,00618	0,0137	1,00

2006 Q4	Exp. weekly return	Std. dev.	Weight
East Capital Baltikumfonden	0,00476	0,0139	0,53
ODIN Norge	0,00779	0,0247	0,12
Morgan Stanley SICAV European Property Fund	0,00552	0,0174	0,10
Gustavia Sverige	0,00674	0,0219	0,08
Länsförsäkringar Fastighetsfond	0,00624	0,0209	0,17
Markowitz portfolio	0,00561	0,0121	1,00

2007 Q1	Exp. weekly return	Std. dev.	Weight
Gustavia Balkan	0,00808	0,0236	0,48
East Capital Östeuropafonden	0,00780	0,0252	0,04
East Capital Rysslandsfonden	0,00804	0,0319	0,04
Carnegie Småbolag	0,00611	0,0225	0,04
Robur Rysslandsfond	0,00737	0,0365	0,40
Markowitz portfolio	0,00771	0,0107	1,00

2007 Q2	Exp. weekly return	Std. dev.	Weight
Gustavia Balkan	0,00795	0,0245	0,25
Länsförsäkringar Fastighetsfond	0,00709	0,0221	0,21
ODIN Norge	0,00707	0,0243	0,12
Credit Suisse Equity Fund (Lux) European Property	0,00554	0,0190	0,09
East Capital Baltikumfonden	0,00427	0,0163	0,33
Markowitz portfolio	0,00624	0,0150	1,00

2007 Q3	Exp. weekly return	Std. dev.	Weight
Gustavia Balkan	0,00832	0,0247	0,27
ODIN Norge	0,00766	0,0243	0,18
East Capital Baltikumfonden	0,00499	0,0166	0,41
ING (L) Invest Latin America	0,00900	0,0312	0,06
Länsförsäkringar Fastighetsfond	0,00646	0,0228	0,08
Markowitz portfolio	0,00673	0,0157	1,00

2007 Q4	Exp. weekly return	Std. dev.	Weight
East Capital Balkanfonden	0,00802	0,0235	0,36
Baring Hong Kong China Fund	0,00937	0,0313	0,12
East Capital Baltikumfonden	0,00499	0,0171	0,39
GAMBAK	0,00862	0,0332	0,13
Markowitz portfolio	0,00708	0,0175	1,00

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The maximum drift portfolio with all stock funds of each period and the Bull portfolio for the same period.

2001 Q4	Exp. weekly return	Std. dev.	Weight
Robur Skogsfond	0,00355	0,0331	0,86
HQ Rysslandsfond	0,00343	0,0545	0,14
Maximum drift portfolio	0,00354	0,0316	1,00
Robur Skogsfond	0,00355	0,0331	1,00
Bull portfolio	0,00355	0,0331	1,00

2002 Q1	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00773	0,0513	1,00
Maximum drift portfolio	0,00773	0,0513	1,00
HQ Rysslandsfond	0,00773	0,0513	1,00
Bull portfolio	0,00773	0,0513	1,00

2002 Q2	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00963	0,0490	1,00
Maximum drift portfolio	0,00963	0,0490	1,00
HQ Rysslandsfond	0,00963	0,0490	1,00
Bull portfolio	0,00963	0,0490	1,00

2002 Q3	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00643	0,0492	1,00
Maximum drift portfolio	0,00643	0,0492	1,00
HQ Rysslandsfond	0,00643	0,0492	1,00
Bull portfolio	0,00643	0,0492	1,00

2002 Q4	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00501	0,0469	1,00
Maximum drift portfolio	0,00501	0,0469	1,00
HQ Rysslandsfond	0,00501	0,0469	1,00
Bull portfolio	0,00501	0,0469	1,00

2003 Q1	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00500	0,0455	1,00
Maximum drift portfolio	0,00500	0,0455	1,00
HQ Rysslandsfond	0,00500	0,0455	1,00
Bull portfolio	0,00500	0,0455	1,00

2003 Q2	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00462	0,0438	1,00
Maximum drift portfolio	0,00462	0,0438	1,00
HQ Rysslandsfond	0,00462	0,0438	1,00
Bull portfolio	0,00462	0,0438	1,00

2003 Q3	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00585	0,0424	1,00
Maximum drift portfolio	0,00585	0,0424	1,00
HQ Rysslandsfond	0,00585	0,0424	1,00
Bull portfolio	0,00585	0,0424	1,00

2003 Q4	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00635	0,0422	1,00
Maximum drift portfolio	0,00635	0,0422	1,00
HQ Rysslandsfond	0,00635	0,0422	1,00
Bull portfolio	0,00635	0,0422	1,00

2004 Q1	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00843	0,0398	1,00
Maximum drift portfolio	0,00843	0,0398	1,00
HQ Rysslandsfond	0,00843	0,0398	1,00
Bull portfolio	0,00843	0,0398	1,00

2004 Q2	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00842	0,0381	1,00
Maximum drift portfolio	0,00842	0,0381	1,00
HQ Rysslandsfond	0,00842	0,0381	1,00
Bull portfolio	0,00842	0,0381	1,00

2004 Q3	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00534	0,0385	1,00
Maximum drift portfolio	0,00534	0,0385	1,00
HQ Rysslandsfond	0,00534	0,0385	1,00
Bull portfolio	0,00534	0,0385	1,00

2004 Q4	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00688	0,0367	1,00
Maximum drift portfolio	0,00688	0,0367	1,00
HQ Rysslandsfond	0,00688	0,0367	1,00
Bull portfolio	0,00688	0,0367	1,00

2005 Q1	Exp. weekly return	Std. dev.	Weight
Länsförsäkringar Fastighetsfond	0,00427	0,0164	1,00
Maximum drift portfolio	0,00427	0,0164	1,00
Länsförsäkringar Fastighetsfond	0,00427	0,0164	1,00
Bull portfolio	0,00427	0,0164	1,00

2005 Q2	Exp. weekly return	Std. dev.	Weight
East Capital Östeuropafonden	0,00538	0,0208	1,00
Maximum drift portfolio	0,00538	0,0208	1,00
East Capital Östeuropafonden	0,00538	0,0208	1,00
Bull portfolio	0,00538	0,0208	1,00

2005 Q3	Exp. weekly return	Std. dev.	Weight
East Capital Östeuropafonden	0,00617	0,0194	1,00
Maximum drift portfolio	0,00617	0,0194	1,00
East Capital Östeuropafonden	0,00617	0,0194	1,00
Bull portfolio	0,00617	0,0194	1,00

2005 Q4	Exp. weekly return	Std. dev.	Weight
ODIN Norge	0,00901	0,0231	1,00
Maximum drift portfolio	0,00901	0,0231	1,00
ODIN Norge	0,00901	0,0231	1,00
Bull portfolio	0,00901	0,0231	1,00

2006 Q1	Exp. weekly return	Std. dev.	Weight
ODIN Norge	0,00883	0,0232	0,33
SKAGEN Kon-Tiki	0,00917	0,0314	0,67
Maximum drift portfolio	0,00906	0,0263	1,00
SKAGEN Kon-Tiki	0,00917	0,0314	1,00
Bull portfolio	0,00917	0,0314	1,00

2006 Q2	Exp. weekly return	Std. dev.	Weight
ODIN Norge	0,01030	0,0216	1,00
Maximum drift portfolio	0,01030	0,0216	1,00
ODIN Norge	0,01030	0,0216	1,00
Bull portfolio	0,01030	0,0216	1,00

2006 Q3	Exp. weekly return	Std. dev.	Weight
ODIN Norge	0,00860	0,0245	1,00
Maximum drift portfolio	0,00860	0,0245	1,00
ODIN Norge	0,00860	0,0245	1,00
Bull portfolio	0,00860	0,0245	1,00

2006 Q4	Exp. weekly return	Std. dev.	Weight
ODIN Norge	0,00779	0,0247	1,00
Maximum drift portfolio	0,00779	0,0247	1,00
ODIN Norge	0,00779	0,0247	1,00
Bull portfolio	0,00779	0,0247	1,00

2007 Q1	Exp. weekly return	Std. dev.	Weight
East Capital Rysslandsfonden	0,00804	0,0319	1,00
Maximum drift portfolio	0,00804	0,0319	1,00
Gustavia Balkan	0,00808	0,0235	1,00
Bull portfolio	0,00808	0,0235	1,00

2007 Q2	Exp. weekly return	Std. dev.	Weight
Gustavia Balkan	0,00795	0,0245	1,00
Maximum drift portfolio	0,00795	0,0245	1,00
Gustavia Balkan	0,00795	0,0245	1,00
Bull portfolio	0,00795	0,0245	1,00

2007 Q3	Exp. weekly return	Std. dev.	Weight
ING (L) Invest Latin America	0,00900	0,0312	0,51
GAMBAK	0,00905	0,0329	0,49
Maximum drift portfolio	0,00902	0,0285	1,00
GAMBAK	0,00905	0,0329	1,00
Bull portfolio	0,00905	0,0329	1,00

2007 Q4	Exp. weekly return	Std. dev.	Weight
Baring Hong Kong China Fund	0,00937	0,0313	1,00
Maximum drift portfolio	0,00937	0,0313	1,00
Baring Hong Kong China Fund	0,00937	0,0313	1,00
Bull portfolio	0,00937	0,0313	1,00

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The AP7 replica portfolio, is the portfolio where we maximize the drift while tracking the risk of the Premium savings fund for each period. The main reason that the difference in variance between the original fund and our replica in not zero is that we are not allowed to choose our weights in fractions of percent.

2001 Q4	Exp. weekly return	Std. dev.	Weight
Robur Skogsfond	0,00355	0,0331	0,33
Pictet Funds(LUX) - Water	0,00242	0,0206	0,58
HQ Rysslandsfond	0,00343	0,0545	0,06
Danske Fonder Sverige Likviditet	0,00076	0,0003	0,03
AP7 replica portfolio	0,00283	0,0220	1,00
AP7 Premium savings fund	-0,00632	0,0219	

2002 Q1	Exp. weekly return	Std. dev.	Weight
Robur Skogsfond	0,00549	0,0311	0,45
HQ Rysslandsfond	0,00773	0,0513	0,27
Moderna Fonder - Sverige Ränta	0,00077	0,0004	0,28
AP7 replica portfolio	0,00478	0,0222	1,00
AP7 Premium savings fund	-0,00265	0,0223	

2002 Q2	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00963	0,0490	0,32
Robur Skogsfond	0,00560	0,0305	0,40
Moderna Fonder - Sverige Ränta	0,00074	0,0004	0,28
AP7 replica portfolio	0,00552	0,0221	1,00
AP7 Premium savings fund	-0,00240	0,0220	

2002 Q3	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00643	0,0492	0,29
Robur Skogsfond	0,00406	0,0295	0,45
Moderna Fonder - Sverige Ränta	0,00075	0,0004	0,26
AP7 replica portfolio	0,00388	0,0221	1,00
AP7 Premium savings fund	-0,00413	0,0212	

2002 Q4	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00501	0,0469	0,44
Morgan Stanley SICAV European Property Fund	0,00018	0,0164	0,36
Moderna Fonder - Sverige Ränta	0,00076	0,0003	0,20
AP7 replica portfolio	0,00242	0,0229	1,00
AP7 Premium savings fund	-0,00501	0,0230	

2003 Q1	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00500	0,0455	0,43
Morgan Stanley SICAV European Property Fund	0,00069	0,0162	0,57
AP7 replica portfolio	0,00255	0,0238	1,00
AP7 Premium savings fund	-0,00400	0,0238	

2003 Q2	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00462	0,0438	0,46
Morgan Stanley SICAV European Property Fund	-0,00024	0,0163	0,54
AP7 replica portfolio	0,00199	0,0241	1,00
AP7 Premium savings fund	-0,00393	0,0242	

2003 Q3	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00585	0,0424	0,47
Morgan Stanley SICAV European Property Fund	0,00071	0,0164	0,53
AP7 replica portfolio	0,00313	0,0239	1,00
AP7 Premium savings fund	-0,00271	0,0243	

2003 Q4	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00635	0,0422	0,56
Moderna Fonder - Sverige Ränta	0,00073	0,0003	0,44
AP7 replica portfolio	0,00388	0,0236	1,00
AP7 Premium savings fund	-0,00226	0,0236	

2004 Q1	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00843	0,0398	0,58
Moderna Fonder - Sverige Ränta	0,00071	0,0003	0,42
AP7 replica portfolio	0,00519	0,0231	1,00
AP7 Premium savings fund	-0,00137	0,0231	

2004 Q2	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00842	0,0381	0,58
Aktia Capital	0,00255	0,0202	0,13
Moderna Fonder - Sverige Ränta	0,00069	0,0003	0,29
AP7 replica portfolio	0,00539	0,0230	1,00
AP7 Premium savings fund	-0,00036	0,0231	

2004 Q3	Exp. weekly return	Std. dev.	Weight
HQ Rysslandsfond	0,00534	0,0385	0,49
Länsförsäkringar Fastighetsfond	0,00223	0,0174	0,51
AP7 replica portfolio	0,00376	0,0226	1,00
AP7 Premium savings fund	-0,00067	0,0227	

2004 Q4	Exp. weekly return	Std. dev.	Weight
Länsförsäkringar Fastighetsfond	0,00399	0,0166	0,50
HQ Rysslandsfond	0,00688	0,0367	0,50
AP7 replica portfolio	0,00544	0,0216	1,00
AP7 Premium savings fund	0,00041	0,0215	

2005 Q1[1]	Exp. weekly return	Std. dev.	Weight
Länsförsäkringar Fastighetsfond	0,00427	0,0164	1,00
AP7 replica portfolio	0,00427	0,0164	1,00
AP7 Premium savings fund	-0,00014	0,0212	

2005 Q2	Exp. weekly return	Std. dev.	Weight
East Capital Östeuropafonden	0,00538	0,0208	0,98
Länsförsäkringar Fastighetsfond	0,00425	0,0167	0,02
AP7 replica portfolio	0,00536	0,0205	1,00
AP7 Premium savings fund	0,00018	0,0205	

2005 Q3	Exp. weekly return	Std. dev.	Weight
East Capital Östeuropafonden	0,00617	0,0194	1,00
AP7 replica portfolio	0,00617	0,0194	1,00
AP7 Premium savings fund	0,00167	0,0194	

2005 Q4	Exp. weekly return	Std. dev.	Weight
Länsförsäkringar Fastighetsfond	0,00696	0,0169	0,12
East Capital Östeuropafonden	0,00805	0,0198	0,36
ODIN Norge	0,00901	0,0231	0,52
AP7 replica portfolio	0,00842	0,0173	1,00
AP7 Premium savings fund	0,00336	0,0174	

2006 Q1	Exp. weekly return	Std. dev.	Weight
East Capital Östeuropafonden	0,00868	0,0205	0,47
ODIN Norge	0,00883	0,0232	0,26
Länsförsäkringar Fastighetsfond	0,00622	0,0166	0,27
AP7 replica portfolio	0,00806	0,0158	1,00
AP7 Premium savings fund	0,00315	0,0158	

[1] In this case there is a single fund that has both higher expected return and lower variance, so our calculations derives at this single fund instead of choosing a portfolio with lower expected return but with the same variance.

2006 Q2	Exp. weekly return	Std. dev.	Weight
Carnegie Småbolag	0,00805	0,0163	0,14
ODIN Norge	0,01030	0,0216	0,30
Länsförsäkringar Fastighetsfond	0,00782	0,0168	0,24
East Capital Östeuropafonden	0,00943	0,0215	0,19
East Capital Baltikumfonden	0,00645	0,0154	0,13
AP7 replica portfolio	0,00872	0,0135	1,00
AP7 Premium savings fund	0,00355	0,0134	

2006 Q3	Exp. weekly return	Std. dev.	Weight
Fondita Nordic Small Cap Placeringsfond	0,00694	0,0193	0,18
ODIN Norge	0,00860	0,0245	0,16
Länsförsäkringar Fastighetsfond	0,00640	0,0205	0,22
East Capital Baltikumfonden	0,00482	0,0156	0,43
Moderna Fonder - Sverige Ränta	0,00035	0,0002	0,01
AP7 replica portfolio	0,00612	0,0136	1,00
AP7 Premium savings fund	0,00251	0,0136	

2006 Q4	Exp. weekly return	Std. dev.	Weight
East Capital Baltikumfonden	0,00476	0,0139	0,41
ODIN Norge	0,00779	0,0247	0,27
Morgan Stanley SICAV European Property Fund	0,00551	0,0174	0,04
Gustavia Sverige	0,00674	0,0219	0,13
Länsförsäkringar Fastighetsfond	0,00624	0,0209	0,15
AP7 replica portfolio	0,00609	0,0137	1,00
AP7 Premium savings fund	0,00270	0,0137	

2007 Q1	Exp. weekly return	Std. dev.	Weight
Gustavia Balkan	0,00808	0,0235	0,97
East Capital Rysslandsfonden	0,00804	0,0319	0,03
AP7 replica portfolio	0,00808	0,0136	1,00
AP7 Premium savings fund	0,00269	0,0136	

2007 Q2	Exp. weekly return	Std. dev.	Weight
Gustavia Balkan	0,00795	0,0245	0,24
Länsförsäkringar Fastighetsfond	0,00709	0,0221	0,23
ODIN Norge	0,00707	0,0243	0,13
East Capital Baltikumfonden	0,00427	0,0163	0,30
Handelsbanken Fonder AB - Lux Ränta	0,00034	0,0002	0,10
AP7 replica portfolio	0,00575	0,0138	1,00
AP7 Premium savings fund	0,00251	0,0138	

2007 Q3	Exp. weekly return	Std. dev.	Weight
Gustavia Balkan	0,00831	0,0247	0,24
ODIN Norge	0,00766	0,0243	0,19
East Capital Baltikumfonden	0,00499	0,0166	0,35
Länsförsäkringar Fastighetsfond	0,00645	0,0228	0,10
Moderna Fonder - Sverige Ränta	0,00034	0,0002	0,12
AP7 replica portfolio	0,00586	0,0136	1,00
AP7 Premium savings fund	0,00297	0,0137	

2007 Q4	Exp. weekly return	Std. dev.	Weight
East Capital Balkanfonden	0,00802	0,0235	0,29
Baring Hong Kong China Fund	0,00937	0,0313	0,10
East Capital Baltikumfonden	0,00499	0,0171	0,31
GAMBAK	0,00862	0,0332	0,11
Handelsbanken Fonder AB - Lux Ränta	0,00037	0,0002	0,19
AP7 replica portfolio	0,00574	0,0140	1,00
AP7 Premium savings fund	0,00278	0,0140	

13 References

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Haugen, R. A (2001) Modern Investment Theory, 5th Ed, Prentice Hall
Premium Pension Authority's homepage: `http://www.ppm.nu/`
Seventh AP fund's homepage: `http://www.ap7.se/`

Fund data and net asset value history available from Premium Pension Authority's homepage.