Kristoffer Vinell: A few inverse and optimal control problems stemming from Torricelli's law

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Sammanfattning

Torricelli's law is often mentioned in the literature at undergraduate level. Because of its generality and elegance, it can be applied to a wide range of outflow problems. However, in most contexts it is used only to describe the flow out of containers with constant cross-sectional area. In this thesis, the applications of Torricelli's law are extended to hold for almost any container of interest from a physical viewpoint.

In particular, we derive an ordinary differential equation that describes the flow from arbitrary containers. The ODE is applied to a few inverse problems, one of which involves finding the proper design for a water clock. In addition, we derive and solve an integral equation concerning flow rates by Laplace transformation.

The ODE is also applied to a few optimal control problems, where the objective is to find minimal emptying times. We use Pontryagin's minimum principle to find necessary conditions for optimality, and prove that an optimal control is of bang-bang type.