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Jens Forsgård: Puiseux parametrizations and invariants of plane algebroid branches

Handledare: Ralf Fröberg

Sammanfattning

The Weierstrass preparation theorem for the ring of germs of holomorphic functions states that any such germ can be, possibly after a linear change of variables, represented as a unit times a Weierstrass polynomial. We give two proofs of the theorem, one using algebra and one using complex analysis. By use of the preparation theorem we then show that plane branches have Puiseux parametrizations, i.e. parametrizations of the form $T \mapsto (T^k, \phi(T))$ where ϕ is analytic in a neighbourhood of the origin, and we give an algorithm to calculate ϕ . We define the multiplicity sequence of an analytic (algebroid) plane branch by use of repeated blowups, and show that it is a topological invariant. By the Puiseux parametrization we can describe an analytic branch as a subring of $\mathbb{C}[[t]]$, and we can then define its value semigroup and characteristic exponents. Either one of them uniquely determines, and is uniquely determined by, the multiplicity sequence. Hence they are also topological invariants. Also, we show how to calculate the semigroup of a plane branch given its parametrization.