

MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik

SJÄLVSTÄNDIGT ARBETE I MATEMATIK

Torsdagen den 18 april kl. 11.00–12.00 presenterar Sarah Alsaadi sitt arbete “The Four Functions Theorem”.

Handledare: Petter Brändén

Plats: Sal 35, hus 5, Kräftriket

Sammanfattning: Let $\mathcal{P}(n)$ be the poset consisting of the subsets of $[n]$ ordered by inclusion and for polynomials $p(x), q(x) \in \mathbb{R}[x_1, x_2, \dots, x_n]$, we put $q(x) \leq p(x)$ if the coefficients of $p(x) - q(x)$ are non-negative. Next we consider functions α, β, γ and $\delta : \mathcal{P}(n) \rightarrow \mathbb{R}_{\geq 0}$ satisfying

$$\alpha(A)\beta(B) \leq \gamma(A \cup B)\delta(A \cap B)$$

for every pair $A, B \in \mathcal{P}(n)$. In this thesis we will study several versions of the four functions theorem [?] and a new one, first noted by Brändén, unpublished:

For each quadruple $\alpha, \beta, \gamma, \delta$ as above and every $A \in \mathcal{P}(n)$ one has

$$\begin{aligned} \left(\sum_{A \in \mathcal{P}(n)} \alpha(A) \prod_{a \in A} x_a \right) \left(\sum_{A \in \mathcal{P}(n)} \beta(A) \prod_{a \in A} x_a \right) &\leq \\ &\leq \left(\sum_{A \in \mathcal{P}(n)} \gamma(A) \prod_{a \in A} x_a \right) \left(\sum_{A \in \mathcal{P}(n)} \delta(A) \prod_{a \in A} x_a \right) \end{aligned}$$

Notice that, since every finite distributive lattice is isomorphic to a sublattice of $\mathcal{P}(n)$ for some n , our main result holds for each finite distributive lattice. In the special case when $x_a = q$ for all a we obtain the q -analogue of the four functions theorem due to Christofides [?] and when $x_a = 1$ for all a we obtain the four functions theorem of Ahlswede and Daykin. In chapter 3, we consider applications to random graphs, linear extensions and to a correlation inequality for certain series weighted by Young tableaux.

Alla intresserade är välkomna!