MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik

SJÄLVSTÄNDIGT ARBETE I MATEMATIK

Torsdagen den 13 mars kl. 10.00–11.00 presenterar Erik Boström sitt arbete "On a Problem of Burgers' Equation with Homogeneous Neumann Boundary Conditions" (15 högskolepoäng, grundnivå).

Handledare: Yishao Zhou

Plats: Sal 32, hus 5, Kräftriket

Sammanfattning: This thesis is concerned with the occurrence of problems corresponding to the numerical treatment of the viscid Burgers' equation together with homogeneous Neumann boundary conditions. It has been shown that the steady state solutions of this system must be constants for an arbitrary initial condition, but for the same problem and for some specific initial conditions, the numerical solutions indeed converge to non-constant steady state solutions. We proved analytically that for arbitrary small non-zero Neumann conditions, the steady states are non-constant and in the shape of a tanh function. Since numerically treated derivatives must be approximated, the homogeneous Neumann conditions are in general approximated by a value up to the size of the machine epsilon of the used floating point format. Thus, these wrong non-constant solutions are existing numerical steady states for the homogeneous problem. It has also been shown that these non-constant steady states are indeed not uniquely defined. For each initial value problem there exist two steady state solutions that mainly depend on the size of the error of the Neumann conditions, the viscosity parameter and the magnitude of the initial condition. The convergence therefore depends on which floating point format we use, since the round off that can occur in the approximation of the Neumann conditions are larger for less accurate formats.
During numerical testing another problem was also found. Most likely it is also caused by the round off errors. Since all constants are steady state solutions, there are no globally defined attractor to the problem, and hence different initial conditions have different steady states. Because of that, small errors that occur in every iteration in the numerical process cannot be cured, since the solution in every iteration can be seen as an initial condition by itself. Hence, in some cases the solution seems to converge to the right steady state, but makes a drastic change to a wrong constant solution. Looking more closely, for an initial condition with an invariant point and for which the steady state solution is the zero constant we can see that the point is shifted by a small value in each iteration. We have approached this problem using an initial condition with a invariant point in which we imposed a Direchlet condition. With this setting the problem was eliminated, which indicates that the errors appear most likely due to the round off errors that occur when the zero value in the point is approximated.

We have also proved that the non-constant solutions does not exist for all numerical schemes. More detailed studies of the impact on specific numerical schemes might therefore be a topic for further studies.

Alla intresserade är välkomna!