MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik

SJÄLVSTÄNDIGT ARBETE I MATEMATIK

Onsdagen den 12 november kl. 11.00–12.00 presenterar Linus Bergqvist sitt arbete "An introduction to Sobolev spaces" (15 högskolepoäng, grundnivå).

Handledare: Joel Andersson

Plats: Sal 22, hus 5, Kräftriket

Sammanfattning: In this thesis we begin by looking at the wave equation with boundary conditions. We find that certain solutions are discarded due to our C2-requirement, although they by any means could be considered to be solutions to our boundary value problem. But if these functions are considered to be solutions to a partial differential equation, in what sense are they differentiable? And in what function space will they lie?

To answer the first question, we introduce the notion of distributions, which can be regarded as a generalization of the concept of a function. For example, the Dirac delta function is not an ordinary function, but it is a distribution. For distributions, we then introduce the notion of weak, or distributional derivative, which is our desired generalization of the usual derivative. To answer the second question we define Sobolev spaces, which are spaces of functions that are sufficiently many times differentiable in the weak sense and whose derivatives all belong to some Lp-space. We first define Sobolev spaces for non-negative integers k, which means that the functions must be k times differentiable in the weak sense. We then extend our definition of Sobolev spaces to arbitrary real numbers. We also define Sobolev spaces for functions defined on the boundary of some open subset of Rn. This can not be done in exactly the same way as for other arbitrary bounded subsets of Rn since the boundary has volume measure 0, and thus all integrals in our usual Sobolev norm become 0. We then derive some results concerning Sobolev spaces. First we define the restriction to the boundary of a function in a Sobolev space, which is not trivial since functions in Sobolev spaces are generally only defined up to a set of measure zero, and thus a function in a Sobolev space can be completely redefined on the boundary without affecting it as an object in a Sobolev space. This restriction map is essential since Sobolev spaces are closely related to partial differential equations with boundary conditions. We then continue by proving that Sobolev spaces are continuously embedded in certain Lp-spaces and Hölder spaces.

Finally, we apply our results regarding Sobolev spaces and distributions to prove a theorem regarding existence and uniqueness of solutions to elliptic boundary value problems.

Alla intresserade är välkomna!