

Abstract

In this thesis, we examine which functions are cyclic with respect to the shift operators in Dirichlet type spaces on the polydisc. That is, we investigate which functions have the property that the only closed subspace that contains the function, which is also invariant under the shift operators, is the entire Dirichlet type space itself.

In particular, we attempt to generalize methods used in the complete characterization of cyclic polynomials in two complex variables to higher dimensions. For example, we generalize a theorem from two variables up to arbitrary dimension, which relates non-vanishing Gaussian curvature of a certain part of the zero set of a function to non-cyclicity of the same function. However, whereas in two variables this theorem was almost always applicable, it turns out that in arbitrary dimension we are not as lucky. Essentially because in two dimensions, the relevant part of the zero set could only be a hypersurface or a finite set, but in higher dimensions there are far more possibilities.

Already in three variables we find a family of polynomials for which the previously mentioned theorem is not applicable, so in the second part of the thesis we attempt to understand the cyclicity properties of this special family. Interestingly enough, it turns out that even for the polynomials on which we could not apply the theorem, we still obtain the same bound on non-cyclicity.

Finally, for the special family of polynomials we develop a method for comparing these polynomials two polynomials in two variables. Using this method we manage to completely understand the cyclicity properties of three variable polynomials in this family whose zero set is either a finite set or a hypersurface, and for polynomials whose zero set is a curve, we show that the cyclicity properties are indeed better than for hypersurfaces, but worse than for finite sets.