Abstract

In this thesis we give an introduction to the dynamics of difference equations and the complex behaviour that sometimes arise out of seemingly simple equations. We start off with a short historical introduction and a comparison with differential equations. We introduce the concept of dynamics of difference equations, where we define and explain concepts such as: orbit, fix points, periodic points, and discuss the notion of stability, as well as state and prove criteria for determining stability of periodic points. We then slowly introduce non-linear dynamics through the example of population models and the logistic map, and we also discuss the theory of bifurcations. We give a short historical introduction to chaotic dynamics, and after making the necessary definitions, we give our definition of chaotic behaviour. Different definitions of chaotic behaviour are discussed, mainly the one due to Devaney, and we briefly address the various ambiguities regarding definitions in this rather recent field of research. After introducing a possible quantification of chaotic behaviour, through the concept of Lyapunov exponents, we move from the dynamics to the geometric aspects of chaotic systems, via fractal geometry. Classical notions of dimension are discussed via e.g. the Lebesgue covering dimension, and with a few examples of fractals, we give some intuition for how and why these classical ideas may be extended to something called fractal dimension. We then give a thorough explanation of different measures of fractal dimension, and apply these ideas to chaotic attractors of dynamical systems, in the form of Renyi dimension. Results of the authors own numerical estimations of the dimension of well known chaotic attractors are presented, and we tie together the dynamics with the geometry side of things with a discussion of the Kaplan-Yorke conjecture. Lastly we give a few concluding remarks and a brief discussion of potential applications to number theory.