Differential forms and Riemannian geometry

- An application to general relativity and gravitational waves

Thomas Aldheimer

Självständigt arbete i matematik, Stockholms Universitet
thomas.aldheimer@gmail.com

August 20, 2017

Abstract

The remarkable theory of general relativity is fundamental for understanding many physical properties of our Universe. The theory connects curvature of 4-dimensional spacetime to gravity. This thesis focuses on the mathematical foundation of curvature, a property of shapes and geometries. The curvature of shapes in Euclidean geometry, i.e. shapes in $\mathbb{R}^3$*, are particularly easy to analyse since* $\mathbb{R}^3$ *has zero curvature. The generalisation of Euclidean geometry is called Riemannian geometry. Useful concepts in Riemannian geometry are defined and derived. Then, using differential forms (multilinear antisymmetric tensor fields), Cartan's structural equations and the Riemann curvature tensor, it is shown how to calculate curvature. This is applied to general relativity and used to illustrate how the existence of gravitational waves can be predicted in theory. Such a prediction was verified in 2016 with the detection of gravitational waves from two merging black holes in a galaxy far, far away.*

Keywords: Cartan's structural equations, differential forms, general relativity, gravitational waves, Riemann curvature tensor, Riemann geometry.