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One-Year Non-Life Solvency II Risk Calculations for Point Process Micro Models: Methods and Applications

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Abstract

The present paper describe how one-year non-life insurance risks according to the Solvency II directive may be calculated consistently for a class of point process based micro reserving models introduced by Norberg (1993). The suggested approach is based on nested simulations. Apart from showing that the simulation based approach is practically feasible, the underlying model is assessed both w.r.t. in-sample and out-of-sample performance, together with an analysis of the simulation error. The methods are applied to real non-life insurance data and the results are put in relation to Solvency II standard formula calculations.

Keywords: Claims reserving, point processes, nested simulations, claims development result, one-year risks

1 Introduction

The current paper is concerned with the calculation of one-year non-life insurance risks as defined by the Solvency II directive with a main focus on the claims reserve risk. This will be done following the marked Poisson process approach introduced in [32]. In [32] the focus is on the theoretical framework describing how to model outstanding claims costs for Reported But Not Settled (RBNS) claims, but also Incurred But Not Reported (IBNR) claims and Not Incurred (NI) claims (actually "covered" and not incurred). The latter claim type corresponds to future not incurred claims stemming from covered contracts, which is closely connected to the Solvency II premium risk which corresponds to future claims from existing contracts as well as to contracts expected to be written over the following 12 months, see [9, Article 105(2)]. The idea of [32] is to

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model the claims generating process at an individual claim level, and then by conditioning on which information that is known the claims reserve, i.e. the Solvency II Best Estimate (BE) reserve, corresponds to the expected value of this future payment process. This modelling approach is what hence forth will be referred to as the "micro model". Other early papers which treat constructive modelling of the claims process are e.g. [3] using martingales and [20] using renewal theory. The ideas laid out in [32] are analysed in e.g. [16, 33, 24, 2], and a textbook treatment may be found in [30]. Another approach is to use techniques closer to compound sums, see e.g. [31, 18, 17]. For more on renewal theory, see e.g. [19, 4, 5] that focus on claim counts and [23] where moments for the entire IBNR claim amount are derived. Other micro level approaches is to use survival analysis techniques, see e.g. [22, 36] as well as more standard GLM regression techniques without any connection to point processes, see e.g. [6, 15].

The focus of the present paper is to describe how to consistently calculate the one-year Solvency II reserve and premium risk for non-life insurance for the micro model as defined in [32, 16, 2]. The one-year reserve risk corresponds to the risk that the claims reserve estimated today re-estimated one year from today will prove insufficient to cover future costs. The forward looking claims reserve loss-distribution calculated today is what is called the Claims Development Result (CDR), and the 99.5% percentile of this loss distribution is what is called the Solvency Capital Requirement (SCR) for non-life reserve risk in the Solvency II directive. Similarly, the one-year Solvency II capital requirement for non-life premium risk is defined analogously to the reserve risk, but for claims that have not yet incurred relating to either already existing contracts or contracts expected to be written during the coming 12 months. Thus, the premium risk is tightly knit to the NI claims of [32]. Further, from a computational perspective the one-year premium risk may be reduced to calculations close to identical with the modelling of IBNR claims. Consequently most focus will be on the one-year reserve risk. Note that this is a very desirable property: once we know how to compute the one-year reserve risk, the one-year premium risk will follow the same procedure, but given other information. This is typically not the case for claims triangle methods such as chain-ladder, see [25], or more recent versions of the chain-ladder such as the double chain-ladder, see [27]. For more on one-year reserve risk and consistency problems with traditional claims triangle methods, see e.g. [34].

The above approach of modelling one-year non-life insurance risks is not standard. The one-year claims reserve risk in the Solvency II directive is based on a version of chain-ladder laid described in [35], see also [14, 34, 7]. Regarding the premium risk, there is no single standard technique, see e.g. [14, 34, 7] and the references therein. As compared with standard claims triangle reserving techniques where the micro model is believed to provide better (ultimo reserve) accuracy, see e.g. [2], the ambition in the present paper is to show the usefulness of the same type of model when calculating one-year re-estimation risks.

In order to be able to calculate one-year re-estimation risks according to the Solvency II directive the model needs to be practically implementable. Parameter fitting will be based on maximum likelihood theory, and the likelihood may be derived following the steps of [16] and [2], based on [21]. This is described in Section 3 and 4. Given fitted parameters, claims characteristics needs to be added to partially known RBNS claims and unknown IBNR (and NI) claims needs to be estimated, which is done using simulation. This procedure will generate future payment processes and the procedure for how this is done follows that of [16] and [2] and is described in Section 5. Once the procedure for generating simulated future payments is available, the one-year reserve and premium risk may be calculated. How this is done, including simulated data updates and parameter re-estimation, is described in Section 6.

The paper ends with a case study based on real insurance data including analysis of model fit as well as out-of-sample performance, see Section 8. Moreover, the one-year risk calculations are based on large scale simulations designed to capture re-estimation risk. This is achieved by using nested simulations – outer (first step BE estimation) and inner (second step BE re-estimation) simulations. Due to this it is of interest to analyse how the so-called computational budget should be used, i.e. to find the relation between number of inner and outer simulations. Such an analysis is carried out in Section 8.5.

The main conclusions of the paper are that the micro model setup for one-year risk calculations are computationally feasible and that the model performance is easy to interpret. Further, compared with the chain-ladder method the case study in Section 8 show that the micro model produces more stable results w.r.t. CDR-distributions together with more accurate BE-reserves.

2 Model outline

As described in the introduction the idea is to focus the modelling on the claims generating process using a marked Poisson process approach introduced by [32, 33] and used in e.g. [16, 2]. Given that such a model can be constructed the claims reserve, i.e. the BE reserve, corresponds to the expected value of the future claims costs produced by the claims generating process. Thus, the key usage of the model is to predict remaining payments from outstanding claims for historical occurrence years. The remaining payments are divided in two parts depending on whether these are stemming from Reported But Not Settled (RBNS) claims or Incurred But Not Reported claims (IBNR), i.e. claims that have occurred, but are yet to be reported to the insurance company.

Every claim will experience the following:

(i) a claim event at some point in time,

(ii) a (possible) reporting delay describing the time between accident and that the insurance company becomes aware of the claim,

(iii) once that the insurer becomes aware of the claim, the handling process will start,

(iv) given that the claim is acknowledged by the insurer there will be a series of payments distributed in time,

(v) the claim is settled.

From a modelling perspective it is usually only interesting to analyse the claims that result in at least one payment. In Section 8.1 we give some comments regarding so-called "zero claims" which are reported, but never will receive any payments. In the present paper the evolution of an actual claim will be described in terms of the following claims characteristics:

1. **Reporting delay** – claims are only known once the reporting delay has been observed, which will separate claims between Reported But Not Settled RBNS and IBNR.

2. **Development process** – after a claim has been reported it can experience a number of different events distributed in time, here we will only consider those relating to payments and/or settlement.

3. **Payments** – the claim events in the development process associated with payments needs to be assigned a (random) payment.

Thus, the above information is all that is needed in order to determine the future costs for already incurred claims. The current modelling approach is hence based on adding the remaining information until settlement for each RBNS claim and given an estimate of the number of IBNR claims add all information needed in order to describe each of these IBNR claims until settlement. In the current paper this will be done using a simulation approach.

Remark 2.1. *Note that once the reporting delay has been added to an IBNR claim the description for how an IBNR claim evolves is identical to that of a RBNS claim.*

In the next section the stochastic model used to describe the claims generating process is defined.

2.1 The claim process as a stochastic model

In the present paper the process according to which claims occurs is assumed to follow a non-homogeneous Poisson process with independent marks. The marks contains information on reporting delay, time points of payments, payment sizes and settlement of claims according to Section 2. For further theoretical background on marked Poisson process, see e.g. [8, 30, 21] as well as [32, 33]. The model presented in the current section is closely connected to the one described in the case study of [2], see also [16].

Given the description of the claim process from Section 2 the corresponding stochastic model that will be used throughout is defined as follows: the claim occurrences of a total portfolio with exposure $w(t)$ is described by a non-homogeneous Poisson process with intensity $\lambda(t)w(t)$ and the time points when claims occur are denoted by T_i. Given that a claim occurs the claim is assigned an independent reporting delay U_i. After that the time U_i has expired three mutually independent (competing) non-homogeneous Poisson processes, that are independent of the occurrence process, are started: one process that generates payments at rate $h_p(t)$, one process that generates settlements at rate $h_{se}(t)$ and one that generates settlements with payments at rate $h_{sep}(t)$. All these processes are stopped as soon as a settlement event has occured. Moreover, the time points between these events for claim i will be denoted by V_{ij} and each of the events generating payments will be assigned a payment of size $P_{ij}(U_i + V_{ij})$, i.e. the size of payments may depend on the time since reporting. To connect this to [2], all information regarding the development process is contained in a random vector of random length denoted by $\mathbf{X}_i$, i.e. each claim is assigned an independent mark $(U_i, \mathbf{X}_i)$.

Remark 2.2. *Throughout we will use upper case letters to denote random variables and lower case letters to denote observations from random variables.*

3 The Likelihood of (partially) observed claims

Regarding the data generating process it follows that observed not yet fully developed claims are observed until some time-point τ, which corresponds to a right censoring. Moreover, in order to observe a claim at all the reporting delay must have been observed. This gives us that the data is also left truncated. For more on censoring and truncation, see e.g. [1].

The full likelihood for the model described in Section 2.1 is a special case of the likelihood given in [2, Eq. (5)], see also [16, Sec. 4.1]. In the current setting we will also assume continuous distribution functions for the time distributions as well as for the payment size distributions. Cumulative distribution functions will be denoted by $F_\bullet(\cdot)$ and density functions as $f_\bullet(\cdot)$. Moreover, we have the following parameters to estimate: λ, i.e. the underlying intensity of the non-homogeneous Poisson process, θ which contains all parameters relating to time distributions and payment distributions together with the hazard functions $h_\bullet(\cdot)$. To stress the dependence on θ we will occasionally write $F_\bullet(\cdot;\theta)$. In [16, 2] the likelihood is described to allow for non-continuous distributions, but given a continuous setting the likelihood reduces to the following expression:

$$
\begin{aligned}
L(\theta, \lambda, h_\bullet) = &\Bigg\{ \left(\prod_{i=1}^{n} \lambda(t_i) F_U(\tau - t_i; \theta) w(t_i) \right) \\
&\times \exp\left(- \int_0^{\tau} \lambda(s) F_U(\tau - s; \theta) w(s) ds \right) \Bigg\} \\
&\times \prod_{i=1}^{n} \frac{f_U(u_i; \theta)}{F_U(\tau - t_i; \theta)} \\
&\times \Bigg\{ \prod_{i=1}^{n} \left(\prod_{j} h_{sep}^{\delta_{ij1}}(v_{ij}) h_{se}^{\delta_{ij2}}(v_{ij}) h_{p}^{\delta_{ij3}}(v_{ij}) \right) \\
&\times \exp\left(- \int_0^{\tau_i} (h_{sep}(s) + h_{se}(s) + h_p(s)) ds \right) \Bigg\} \\
&\times \prod_{i=1}^{n} \prod_{j'} f_{P|U_i + V_{ij'} = u_i + v_{ij'}}(x_{ij'}; \theta), \qquad (1)
\end{aligned}
$$

where δ_{ijk} denote the indicator function belonging to claim i defined by

$$
\delta_{ijk} := \begin{cases} 1 & \text{if } j = k, \\ 0 & \quad j \neq k \end{cases}
$$

where $k = 1, 2, 3$ corresponds to claim type sep, se and p, respectively, and where $\tau_i = \min(\tau - t_i - u_i, v_i)$.

Before proceeding further it is worth to comment on (1). The first two lines in the likelihood relates to observed claim events, i.e. claims occur following a non-homogeneous Poisson process with intensity $\lambda(t)$ out of which we only observe the claims for which $T_i + U_i \leq \tau$, which explains the thinning w.r.t. $F_U(\tau - t_i; \theta)$. That is, claim events are observed at rate $\lambda(t) F_U(\tau - t; \theta) w(t)$ and the exponential expression corresponds to the probability that the claim did not occur until time t given the rate function, i.e. the claim has "survived" up to the time immediately prior to t.

The third line of (1) relates to the estimation of the delay distribution governing U_i which is depending on that data is left-truncated. That is, we can not observe U_i unless $T_i + U_i \leq \tau$, which gives us the conditional density of U_i given $t_i + U_i \leq \tau$, since T_i is observed. For more on this, see Section 5.2.

Lines four and five are analogous to lines one and two, but w.r.t. when payments, payments with settlements and settlements without payments occurs, assuming that these three types of events are mutually independent and independent of λ and the delay.

Finally, the last line corresponds to size of payments which may depend on time since reporting.

Further, one can also note that (1) may be reduced further, since the first three lines contains factors $F_U(\tau - t_i;\theta)$ that cancel:

$$
\begin{aligned}
L(\theta, \lambda, h_\bullet) = & \left(\prod_{i=1}^{n} \lambda(t_i) f_U(u_i;\theta) w(t_i) \right) \\
& \times \exp\left(- \int_0^{\tau} \lambda(s) F_U(\tau - s;\theta) w(s) ds \right) \\
& \times \prod_{i=1}^{n} \left(\prod_{j} h_{sep}^{\delta_{ij1}}(v_{ij}) h_{se}^{\delta_{ij2}}(v_{ij}) h_{p}^{\delta_{ij3}}(v_{ij}) \right) \\
& \times \exp\left(- \int_0^{\tau_i} (h_{sep}(s) + h_{se}(s) + h_p(s)) ds \right) \\
& \times \prod_{i=1}^{n} \prod_{j'} f_{P|U_i+V_{ij'}=u_i+v_{ij'}}(x_{ij'};\theta).
\end{aligned} \tag{2}
$$

Remark 3.1. *Note that even though the claim occurrence process is independent of its marks, we will still only observe partial information about the underlying claim occurrence process due to right censoring and left-truncation. The effect of this is that we will only be able to observe the thinned claim occurrence process. An artefact of this observation is that we can not estimate the delay distribution and $\lambda(t)$ separately.*

4 Estimation of model parameters and parameter uncertainty

The likelihood from (2) is a rather general object. In the case study described in Section 8 we need to make various assumptions regarding distributions and functional forms of the intensity functions. Due to data disclosure and availability of data we will in Section 8 limit the intensity functions to being piecewise constant.

That is, let $[d_{l-1}, d_l)$ where $l = 1, ..., m$ be a partition of the time interval $[0, \tau]$ where $\lambda(t)$ and $w(t)$ are assumed to be piecewise constant and let $[\tilde{d}_{l-1}, \tilde{d}_l)$ where $l = 1, ..., \tilde{m}$ be a partition of the time interval $[0, \tau]$ where the intensities $h_e(t), e \in \{p, s, sep\}$, are assumed piecewise constant, where

$$
\begin{aligned}
\lambda(t) &\equiv \lambda_l,\ t \in [d_{l-1}, d_l),\ l = 1, \dots, m, \\
w(t) &\equiv w_l,\ t \in [d_{l-1}, d_l),\ l = 1, \dots, m, \\
h_e(t) &\equiv h_{e,l},\ e \in \{p, s, sep\},\ t \in [\tilde{d}_{l-1}, \tilde{d}_l),\ l = 1, \dots, \tilde{m}.
\end{aligned}
$$

Further, let n_l^{oc} denote the number of observed claims in interval l and let $n_{e,l}^{oc}$ denote the number of observed events of type $e \in \{p, s, sep\}$ in interval l.

By using the above notation the likelihood from (2) may be re-written according to the following:

$$
\begin{aligned}
L(\theta, \lambda, h_\bullet) = &\left(\prod_{i=1}^{m} (\lambda_i w_i)^{n_i^{oc}}\right) \exp\left(-\sum_{i=1}^{m} \lambda_i w_i \int_{d_{i-1}}^{d_i} F_U(\tau - s; \theta) ds\right) \\
&\times \prod_{i=1}^{\tilde{m}} \left(h_{sep,i}^{n_{sep,i}^{oc}} h_{se,i}^{n_{se,i}^{oc}} h_{p,i}^{n_{p,i}^{oc}}\right) \\
&\quad \times \exp\left(-\sum_{i=1}^{n} \sum_{j=1}^{\tilde{m}} (h_{sep,j} + h_{se,j} + h_{p,j}) \int_{\tilde{d}_{j-1}}^{\tilde{d}_j} 1_{\{s \geq \tau_i\}} ds\right) \\
&\times \prod_{i=1}^{n} f_U(\tau - t_i; \theta) \prod_{j'} f_{P|U_i + V_{ij'} = u_i + v_{ij'}}(x_{ij'}; \theta). \qquad (3)
\end{aligned}
$$

All parameters $\{\lambda_l\}, \{h_{\bullet,l}\}$ and θ are then estimated using standard maximum likelihood theory. In particular it is worth noting that the likelihood for $\{h_{\bullet,l}\}$, may be factored out and treated separately, resulting in the following ML-estimator

$$
\hat{h}_{e,l} = \frac{n_{e,l}^{oc}}{\sum_{i=1}^{n} \int_{\tilde{d}_{l-1}}^{\tilde{d}_l} 1_{\{s \geq \tau_i\}} ds}, \; e \in \{p, se, sep\}. \qquad (4)
$$

Note that payment size distributions are also independent of all other parameters and may be analysed separately.

Regarding the reporting delay distributions and $\{\lambda_l\}$, these can not, as seen above, be treated separately, see Remark 3.1. From [2] it is not entirely clear how they address this topic, see the discussion concerning Eq. (8) in [2, Sec. 4.2]. It is however straightforward to maximise the parameters in θ that relates to the reporting delay distribution simultaneously with the λ_ls. This is what is done in Section 8. Note that this dependence between the parameter estimators will affect their joint parameter uncertainty.

Concerning parameter uncertainty and re-estimation risk, this will tend to become computationally heavy. Following the lines of [2, Sec. 5.3] we make use of that

$$
(\{\hat{\lambda}_l\}, \{\hat{h}_{e,l}\}, \hat{\theta}) \sim \text{asym. N}((\{\hat{\lambda}_l\}, \{\hat{h}_{e,l}\}, \theta), \hat{\mathbf{C}}),
$$

where $\hat{\mathbf{C}} = \widehat{Cov}(\{\hat{\lambda}_l\}, \{\hat{h}_{e,l}\}, \hat{\theta}) = \mathbf{I}^{-1}(\{\hat{\lambda}_l\}, \{\hat{h}_{e,l}\}, \hat{\theta})$, where $\mathbf{I}^{-1}(\cdot)$ corresponds to the inverse of the observed Fisher information obtained from the numerical estimation procedure used to maximise the likelihood.

Remark 4.1. *All hazard rates are mutually independent and independent of all other model parameters. Thus, based on* (3) *and* (4) *it follows that*

$$
\widehat{Var}(\hat{h}_{e,l}) = \frac{1}{-\frac{\partial^2}{\partial h_{e,l}^2} \log L(h_{e,l})|_{h_{e,l} = \hat{h}_{e,l}}} = \frac{n_{e,l}^{oc}}{\left(\sum_{i=1}^{n} \int_{\tilde{d}_{l-1}}^{\tilde{d}_l} 1_{\{s \geq \tau_i\}} ds\right)^2}. \qquad (5)
$$

Remark 4.2. *Since the hazard rates are assumed to be piecewise constant, these are estimated independently of each other, making the covariance matrix in the asymptotic normal distribution diagonal. This means that any pattern seen in the point estimates of the hazard rates when looked upon jointly may be destroyed when adding parameter uncertainty using the asymptotic normal distribution assumption if* (5) *becomes too large.*

5 Simulating claims characteristics

The stochastic model described in Section 2.1 define how we believe that claims data is generated over time. When it comes to reserving we are interested in the value of the future payments generated by already incurred claims. For RBNS claims we already have observed partial information, in particular we have $T + U \leq t$, and it hence amounts to simulating the future evolution of the development process. Regarding the IBNR claims we know that these must satisfy $T + U > t$. The first step is hence to simulate the number of IBNR claims, and given the number of IBNR claims these need to be distributed in time in such a way that $T + U > t$. Once this is done the remaining unknown information corresponds to that of a RBNS claim that has been observed exactly at $T+U$. Given this, the simulation of the evolution of the development process is identical for RBNS and IBNR claims.

In Section 5.1-5.3 we describe in detail how the simulation is carried out. This closely follows the procedure from [2, Sec. 5], see also [16, Sec. 4.3].

5.1 Simulating number of IBNR claims

Based on the model from Section 2.1 together with the assumptions on piecewise constant intensities from Section 4 it follows that the number of IBNR claims stemming from the time interval $[d_{i-1}, d_i)$, denoted by $N_{IBNR,i}$, is given by

$$N_{IBNR,i} \sim \text{Po}\left(\hat{\lambda}_i w_i \int_{d_{i-1}}^{d_i} (1 - F_U(\tau - t; \hat{\theta}))dt\right). \tag{6}$$

This is reasonable, since the thinning $1-F_U(\tau-t;\hat{\theta})$ is just another way of saying that the claims have "survived" detection during the time interval $[d_{i-1}, d_i)$.

5.2 Simulating IBNR time of accident and reporting delay

For each simulated IBNR claim we know that the time of accident, T_i, and reporting delay, U_i, must satisfy $T_i + U_i > \tau$. Thus, we want to draw pairs (T_i, U_i) from

$$\begin{aligned} P(T \leq t, U \leq u | T + U > \tau) = {} & P(U \leq u | T + U > \tau, T \leq t) \\ & \times P(T \leq t | T + U > \tau). \end{aligned}$$

Further, since claims occur according to a Poisson process it follows that the marginal distribution of accident time given that a claim has occurred within a certain time interval is uniform, i.e. $T_i \sim \text{Un}(0, c)$, say, where $c > 0$ depends on the time interval decomposition of the claim year w.r.t. λ. Moreover, by definition T and U are independent. Hence, by combining these facts and using standard conditioning it follows that

$$f_{T|T+U>\tau}(t) = \frac{P(U > \tau - t)}{\int_0^c P(U > \tau - s)ds}, \tag{7}$$

and

$$f_{U|T+U>\tau,T=t}(u) = f_{U|U>\tau-t}(u) = \frac{f_U(u)}{P(U > \tau - t)}. \tag{8}$$

Consequently, following the above, you start by drawing $T_i|T_i + U_i > \tau$ from (7), and given $T_i = t_i$, you draw a U_i from (8).

5.3 Simulating the development process for RBNS and IBNR claims

We will now describe how the development process is simulated, but first let s_i denote the time since reporting for claim i. Further, in order to simplify the exposition below we return to using a continuous $h_e(t)$ even though these rates are assumed piecewise constant.

The development process for claim i is simulated according to the following:

(i) The time until the next event is drawn from

$$P(V \leq v|V > s_i) = \frac{\exp\left(-\int_0^{s_i} \sum_e h_e(s)ds\right) - \exp\left(-\int_0^{v} \sum_e h_e(s)ds\right)}{1 - \exp\left(-\int_0^{s_i} \sum_e h_e(s)ds\right)}.$$

(ii) Given the next event time v_i the probability that the next event is of type e is given by

$$\frac{h_e(v_i)}{\sum_e h_e(v_i)}$$

due to the Poisson structure and where $e \in \{p, se, sep\}$.

(iii) Given that the event was of type $\{p, sep\}$ a payment is drawn from $F_P(u_i + v_i)$.

(iv) Set $s_i = u_i + v_i$.

(v) If $e \notin \{se, sep\}$ go to Step (i) otherwise stop.

5.4 Parameter uncertainty

In order to account for parameter uncertainty we start each simulation by drawing parameters from the asymptotic normal distribution described at the end of Section 4 in Remark 3.1.

6 Best estimate reserves for RBNS and IBNR claims and one-year insurance risks

The procedure laid out in Section 5.1-5.3 defines how to simulate the necessary claims characteristics needed in order for the evolution of all RBNS and IBNR claims to become known until final settlement. From the result of each such simulation carried out at time t it is straight forward to sum up all payments stemming from RBNS and IBNR claims in (t, ∞), which corresponds to one simulated claims reserve. Thus, by repeating the steps from Section 5.1-5.3 κ times we will obtain the empirical simulated distribution of the total outstanding claims reserve. Let $R_{RBNS,k}(t)$ denote the sum of all RBNS related payments in simulation $k, k = 1, \ldots, K$, that are generated by claims that have occurred up to and including time t, and define $R_{IBNR,k}(t)$ analogously. The total reserve in simulation k is then given by

$$R_k(t) = R_{RBNS,k} + R_{IBNR,k}, \; k = 1, \ldots, \kappa. \tag{9}$$

Let

$$\mathcal{R}(t) := \{R_k(t); k = 1, \ldots, \kappa\}.$$

The Best Estimate (BE) reserve according to the Solvency II-directive, see [9, Article 77(2)], is then given by:

$$\widehat{BE}(t) := \hat{E}[\mathcal{R}(t)] = \frac{1}{\kappa} \sum_{k=1}^{\kappa} R_k(t), \tag{10}$$

i.e. $\widehat{BE}(t)$ is an estimate of expected value of all outstanding payments in (t, ∞).

Remark 6.1. *Note that $\mathcal{R}(t)$ is defined w.r.t. all available information up to and including time t. Consequently $\mathcal{R}(t)$ has an implicit dependence on the natural filtration generated by the micro model generated up to time t. This fact becomes important in the next sub-section when the one-year re-estimation risk of reserves will be addressed.*

Remark 6.2. *The above BE reserve is un-discounted and does not account for inflation. Further, the expense reserve is not included in the above reserve estimate.*

6.1 One-year reserve risk

The Solvency II-directive requires that the re-estimation risk shall be quantified, and in particular the one-year re-estimation risk. Ideally this is captured according to the following: let $C_k(t+1)$ denote the amount of payments that are realised (paid out) in simulation $k, k = 1, \ldots, \kappa$, during $(t, t+1]$. Further, let $\widehat{BE}_k^{\nu}(t+1)$ denote the best estimate reserve for the outstanding claims in $(t+1, \infty)$ in simulation k given the additional information that has become known in $(t, t+1]$ when ν inner simulations are used. That is, simulated IBNR claims become known according to the model, RBNS claims get updated claims characteristics, parameters are re-estimated etc. The realised one-year reserve result in simulation k is then given by

$$D_k^{\nu}(t+1) := \widehat{BE}_k^{\nu}(t+1) + C_k(t+1) - \widehat{BE}(t),$$

where $D_k^{\nu}(t+1) > 0$ corresponds to a loss. Continuing, let

$$\mathcal{D}_{\kappa}^{\nu}(t+1) := \{D_k^{\nu}(t+1); k = 1, \ldots, K\}. \tag{11}$$

The one-year Solvency Capital Requirement (SCR) for reserve risk according to the Solvency II-directive is then given by

$$\widehat{SCR}_R(t) := \hat{q}_{0.995}(\mathcal{D}_{\kappa}^{\nu}(t+1)), \tag{12}$$

where $\hat{q}_{\alpha}(\cdot)$ corresponds to the empirical α-quantile function, i.e. the $100\alpha\%$ Value-at-Risk, see [9, Article 101(3)]. The above presentation is in alignment with [34].

Remark 6.3. *The quantity $D_k^{\nu}(t+1)$ is, possibly up to a change of sign, what is often referred to as the Claims Development Result (CDR).*

Remark 6.4. *Each $\widehat{BE}_k^{\nu}(t+1)$ is based on an additional ν inner simulations in accordance with the procedure described in order to calculate* (10)*. The question of choosing a suitable ν (and κ) will be addressed in Section 8.5 below.*

Remark 6.5. *Note that the above procedure for calculating the SCR for reserve risk is based on that the information that becomes realised during $(t, t+1]$ is used to re-estimate parameters etc. In practice this becomes computationally very heavy. As an approximation all simulated IBNR claims that become known in $(t, t+1]$ are transferred to RBNS claims and updated accordingly. Further, as described in Section 5, each outer simulation k has a collection of simulated parameters assigned to it which are drawn from the asymptotic ML-error distribution. These parameters are kept constant when estimating the corresponding $\widehat{BE}_k^{\nu}(t+1)$ connected to the evolution of claims payments in $(t+1, \infty)$.*

Remark 6.6. *Given* (11) *it is straight forward to calculate other metrics of one-year reserve risk such as Expected Shortfall.*

6.2 Premium risk

The one-year premium risk according to the Solvency II-directive corresponds to the risk that the cost from next years written polices (including renewals) and costs from unexpired contracts will be larger than expected. This is analogous to the one-year reserve risk except that you at the end of year t set a reserve for the claims that you *expect to occur during* year $t+1$. Thereby, given an estimate today, at t, of next years premium earnings in $(t, t+1]$, $\tilde{P}(t)$, the one-year premium result in simulation k is given by

$$G_k^\nu(t+1) := \widetilde{\widetilde{BE}}_k^\nu(t+1) + \tilde{C}_k(t+1) - \tilde{P}(t), \tag{13}$$

where $\widetilde{\widetilde{BE}}_k^\nu(t+1)$ corresponds to the BE reserve for the claims that have occurred *during* year $t+1$ and $\tilde{C}_k(t+1)$ denotes the corresponding payments in $(t, t+1]$ and ν denotes the number of inner simulations used. As for the one-year reserve risk $G(t+1) > 0$ corresponds to a loss. The premium risk defined by (13) is again in line with [34]. Analogously to the one-year reserve risk we let

$$\mathcal{G}_\kappa^\nu(t+1) := \{G_k^\nu(t+1); k = 1, \ldots, \kappa\},$$

which gives us the one-year SCR for premium risk according to

$$\widehat{SCR}_P(t) := \hat{q}_{0.995}(\mathcal{G}_\kappa^\nu(t+1)).$$

Remark 6.7. *Again, in agreement with the one-year reserve risk, Remark 6.5 is valid for premium risk.*

Remark 6.8. *At the end of year t all claims that eventually will occur during year $t+1$ may from the above simulation procedure's perspective be treated as IBNR-claims. That is, for occurrence (accident) year $t+1$ we can again make use of the simulation procedure from Section 5.1-5.3, but where there are only IBNR claims which are given by*

$$N_{IBNR}(t+1) \sim Po\left(w_{t+1}\hat{\lambda}_{t+1}\right), \tag{14}$$

where w_{t+1} and $\hat{\lambda}_{t+1}$ are suitably chosen.

Again, note that this is in close connection to "Not Incurred" (NI) claims discussed in [32, 33].

Remark 6.9. *Operating expenses are not included in the above definition of premium risk unless these are non-random and may be included in $\tilde{P}$.*

7 The chain-ladder method

In Section 8 we want to analyse the performance of the micro-model in relation to real data. Further, in order to set the performance of the micro-model in perspective we will use a version of the classical chain-ladder method as reference. In particular an implementation of the chain-ladder with bootstrapping is used to model one-year risks. The bootstrapping used is the one described in [12]. For more on other stochastic versions of the chain-ladder method, see e.g. [11, 12] and the references therein.

7.1 One-year Reserve Risk

As in Section 6.1 we follow the definition of one-year reserve risk given by [34]. Again, we want to produce $\mathcal{D}(t+1)$ from (11) in order to be able to calculate $\widehat{SCR}_R(t)$ from (12).

In [34] several possibilities are suggested on how to simulate new chain-ladder diagonals. One of the prosed methods is to simulate next years payment from a log-normal distribution with a mean given by the chain-ladder estimate and the variance given by Mack's standard distribution free chain-ladder conditional variance assumption.

Given this model diagonals are simulated where after the chain-ladder development factors are re-estimated which are then used to produce new reserve estimates.

7.2 One-year premium risk

Regarding premium risk for the chain-ladder method it is not directly applicable in the same way as described in Section 6.2. This is a consequence of that the chain-ladder method is based on that there are at least some partially observed claims available. When it comes to premium risk there are not yet any observed claims for next years claims. The classical chain-ladder method lacks an initiation step for not yet incurred claims. Due to this we will not make any chain-ladder comparison w.r.t. premium risk. For more on premium risk and chain-ladder see e.g. [34].

8 Case study

The previous sections have been devoted to explaining the necessary background theory needed to calculate one-year non-life insurance risks in accordance with the Solvency II-directive. We now proceed with a case study based on real data. For the purpose of this study it is not relevant to know more about the type of business other than it is non-life insurance.

8.1 Data

The data consists of claim payments for 9 consecutive years containing approximately 11 000 unique claims. The claims can be settled directly as well as receive multiple payments before being settled. In total the dataset contains approximately 22 000 events. In addition there are approximately 4 000 zero claims, i.e. claims that never received any payments before settlement. These zero claims need to be accounted for. This is done by adding an extra step in the simulation which corresponds to a thinning of potential IBNR claims before initialising their development processes, where the thinning is done using a logistic regression model.

Further, all results have been normalised due to reasons of confidentiality. This is described in more detail below.

8.2 Trends in the data

We will now analyse the underlying data in more detail and comment on trends. As a first step we start by looking at patterns seen in total cumulative payments and counts in terms of development factors per occurrence year. This is relevant, since the chain-ladder method will be used as reference method below.

The development factors are summarised in Table 1.

Remark 8.1. *In Table 1 as well as in the tables below, the exposure per occurrence year has been normalised against the outcome of development year 1. The actual exposure during the analysed time period has, however, changed substantially. Due to confidentiality we do not want to discuss this further, and for the purpose of identifying other trends in data this information is not needed.*

From Table 1 it is seen that there are indications of that more payments are made during development year 1 for later occurrence years. This fact is a consequence of an improved claims handling process which has decreased the the claims handling times. This may also have a slight effect on the development between year 2 and 3. Regarding the development of claim counts, see Table 2, the pattern is rather homogeneous, except for that there is an indication of that claims are reported earlier for later occurrence years. This suggests that there may be either a trend in the delay distribution or that the pattern for how claims are reported during the claim year may have changed. Moreover, from Table 2 it is evident that there is a non-negligible fraction of IBNR claims up until at least development year 5.

Table 1: The development factors per occurrence year based on cumulative payments.

	Development year							
Occurrence year	2	3	4	5	6	7	8	9
x	2.82	1.70	1.28	1.06	1.08	1.00	1.06	1.02
$x+1$	2.62	1.86	1.23	1.16	1.07	1.04	1.02	
$x+2$	2.89	1.76	1.19	1.10	1.06	1.02		
$x+3$	2.35	1.66	1.21	1.13	1.04			
$x+4$	1.89	1.50	1.31	1.08				
$x+5$	2.26	1.57	1.17					
$x+6$	2.43	1.47						
$x+7$	1.91							

Continuing, in Table 3 the empirical mean payments for each occurrence year and reporting year are summarised. All mean payments are normalised w.r.t. the mean from the first development year for each accident year. From Table 3 we see indications of an increasing trend as a function of time since reporting. Further, we see no indications of trends across occurrence years.

Table 2: The development factors per occurrence year based on cumulative claim counts.

	Development year							
Occurrence year	2	3	4	5	6	7	8	9
x	1.51	1.10	1.05	1.03	1.01	1.00	1.01	1.00
$x+1$	1.45	1.13	1.04	1.02	1.01	1.00	1.00	
$x+2$	1.51	1.12	1.06	1.02	1.02	1.01		
$x+3$	1.43	1.12	1.06	1.02	1.01			
$x+4$	1.43	1.10	1.07	1.02				
$x+5$	1.40	1.11	1.05					
$x+6$	1.41	1.10						
$x+7$	1.38							

Table 3: The mean of the payments depending on occurrence year and development year. The payments have been normalised by the mean from the first reporting year of occurrence year x.

	Reporting year								
Occurrence year	1	2	3	4	5	6	7	8	9
x	1.00	1.76	3.4	3.55	1.76	6.54	1.17	6.96	2.56
$x+1$	0.92	2.05	3.49	3.57	5.52	4.37	5.43	2.69	
$x+2$	1.22	2.35	3.7	3.1	4.7	4.02	3.12		
$x+3$	1.32	2.06	3.29	2.81	4.47	3.07			
$x+4$	1.61	1.96	2.72	3.58	3.59				
$x+5$	1.16	2.19	3.1	2.85					
$x+6$	1.09	2.06	2.74						
$x+7$	1.6	2.34							
$x+8$	1.25								

Table 4 shows the relative changes in the hazard rates. The hazard rates are estimated for each individual occurrence year, but we focus on the first 5 development periods, where the length of a development period is 60 days. The actual hazard rates that will be used in the model will be sub-divided into 20 intervals of length 60 days, see Section 8.4. The hazard rates are normalised by the corresponding hazard rates from occurrence year x and the first hazard rate period. From Table 4 there is indication of that the hazard rates for settled with a payment events have increased. This is in alignment with the decrease in time for claims handling commented on above. For the other types of events no signs of trends are detected.

Another interesting characteristic of the dataset being analysed is the dynamics of number of open and closed claims as a function of occurrence and development year. This is summarised in Table 5. From Table 5 we see significant changes over occurence years. In particular, more claims are settled earlier, which also is an artefact of the improved claims handling process.

8.3 Results

We start by stating specific considerations made in the modelling together with a summary of fitted parameter values, see Section 8.4. Further, given model assumptions and fitted parameters we analyse the number of simulations needed in order for the BE reserve and SCR to converge, see Section 8.5. Given this we proceed to verify the model fit by comparing in-sample performance of the model w.r.t. RBNS and IBNR payments, see Section 8.6. In Section 8.7 the out-of-sample performance of the model's BE reserve and SCR are compared with corresponding results for the chain-ladder implementation described in Section 7.

8.4 Model assumptions and estimation

We will now give a brief account of the results of the fitting procedures that have been carried out together with essential model assumptions. The different components of the stochastic model are treated as follows:

1. **The delay distribution**
   Comparing standard continuous distributions using AIC suggests that the delay distribution may be modelled using a log-normal distribution. Due to the heavy tail of the log-normal distribution we apply a 30 year cap on the reporting delay. Model parameters together with confidence intervals are shown in Figure 1.

2. **Intensities**
   The intensities λ are assumed piecewise constant on yearly intervals. The ratio of the means $\hat{\lambda}_i/\hat{\lambda}_1$ are displayed in Figure 3a together with 95% confidence intervals.

3. **The payment distribution**
   An analysis based on standard continuous distributions together with AIC suggests that the size of payments as a function of time since reporting may be modelled using log-normal distributions. The payment size distributions are assumed constant on intervals of 365 days, and they do not change after year five since reporting. Fitted parameters together with their respective estimation uncertainty is shown in Figure 2.

4. **Hazard rates**
   The hazard rates are assumed to be constant on intervals of 60 days. After

Table 4: Relative changes in the hazard rates. The hazard rates are estimated with data from the different occurrence years and here we assume no further changes in the hazard rates after 5 intervals. The hazard rates are normalised with $h_{e,1}$ estimated on occurrence year x.

		Hazard rate period				
Type	Occurrence year	1	2	3	4	5
Se	x	1.00	2.04	1.94	1.41	4.98
	$x+1$	1.18	1.95	1.19	1.92	5.14
	$x+2$	1.38	1.51	2.05	3.15	5.06
	$x+3$	1.50	1.69	0.83	1.57	4.65
	$x+4$	1.08	1.32	1.15	1.19	4.18
	$x+5$	0.61	1.89	2.87	1.54	4.54
	$x+6$	0.89	2.55	3.34	1.37	4.28
	$x+7$	1.95	6.56	5.04	3.49	2.93
	$x+8$	4.00	16.89	14.06	4.38	6.54
Sep	x	1.00	0.14	0.08	0.02	0.11
	$x+1$	0.84	0.13	0.08	0.03	0.11
	$x+2$	0.92	0.12	0.06	0.08	0.13
	$x+3$	1.42	0.20	0.12	0.06	0.12
	$x+4$	1.83	0.18	0.09	0.06	0.10
	$x+5$	2.22	0.21	0.07	0.03	0.11
	$x+6$	2.32	0.29	0.14	0.06	0.11
	$x+7$	3.53	0.37	0.20	0.11	0.10
	$x+8$	6.57	0.91	0.40	0.22	0.13
P	x	1.00	0.41	0.19	0.09	0.13
	$x+1$	1.06	0.36	0.16	0.11	0.12
	$x+2$	0.96	0.37	0.15	0.08	0.12
	$x+3$	1.00	0.41	0.18	0.13	0.14
	$x+4$	0.99	0.38	0.18	0.09	0.11
	$x+5$	0.98	0.35	0.19	0.11	0.10
	$x+6$	0.84	0.47	0.22	0.12	0.10
	$x+7$	0.95	0.39	0.17	0.09	0.10
	$x+8$	0.83	0.57	0.30	0.10	0.10

Table 5: The proportion of settled/open claims relative the number of settled/open claims at the end of development year 1 for each occurrence year.

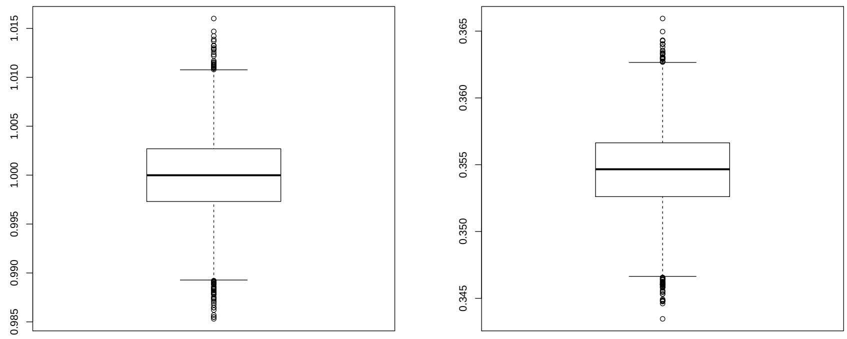
	Development year							
Occurrence year	2	3	4	5	6	7	8	9
				Settled claims				
x	1.72	1.95	0.93	0.34	0.29	0.43	0.13	0.17
$x+1$	1.44	1.98	0.82	0.40	0.51	0.16	0.15	
$x+2$	2.13	1.79	1.31	0.61	0.67	0.19		
$x+3$	0.79	1.35	0.86	0.41	0.29			
$x+4$	0.94	0.85	1.11	0.46				
$x+5$	0.67	0.70	0.88					
$x+6$	0.68	0.89						
$x+7$	0.75							
				Open claims				
x	0.65	0.34	0.19	0.15	0.11	0.04	0.03	0.01
$x+1$	0.71	0.33	0.22	0.17	0.06	0.03	0.02	
$x+2$	0.66	0.38	0.22	0.14	0.03	0.02		
$x+3$	0.83	0.43	0.21	0.12	0.04			
$x+4$	0.82	0.58	0.25	0.12				
$x+5$	0.82	0.54	0.16					
$x+6$	0.80	0.34						
$x+7$	0.60							

20 intervals we assume no further changes in the hazard rates, i.e. after roughly 3 years the hazard rates are constant. The fitted hazard rates together with 95% confidence intervals are shown in Figure 3.

At this point we will not go further into details about fit, but will return to this in Section 8.6 where we verify that the above proposed model capture the in-sample dynamics of payments per occurrence year and development year well. This is a deliberate choice in order to avoid overfitting, which is especially important since the main application of the model is to perform out-of-sample predictions, the latter being analysed in more detail in Section 8.7.

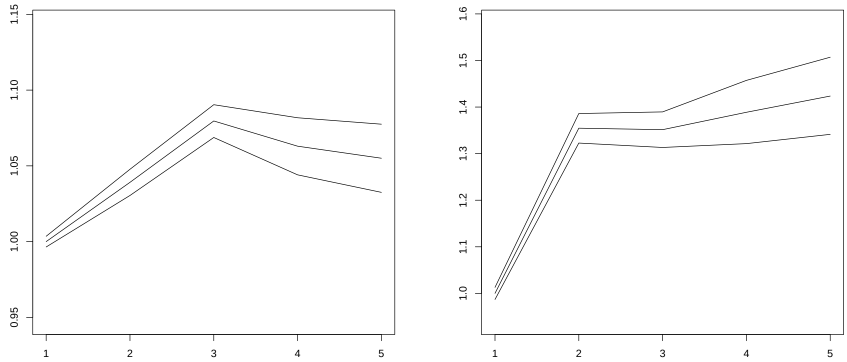
Remark 8.2. *Note that in the simulation procedure described in Section 5 all parameters are re-sampled from their asymptotic estimation error distributions, i.e. according to the uncertainty shown in Figure 1-3. Further, again we stress that the parameters are re-sampled with dependencies corresponding to the ones implied by the fitting procedure, e.g.* λ *will depend on the parameters in the delay distribution etc. For more on this we refer to Section 4.*

Remark 8.3. *By assuming that the hazard rates are constant beyond approxi-*



(a) Mean parameter on log-scale (b) Standard deviation parameter on log-scale

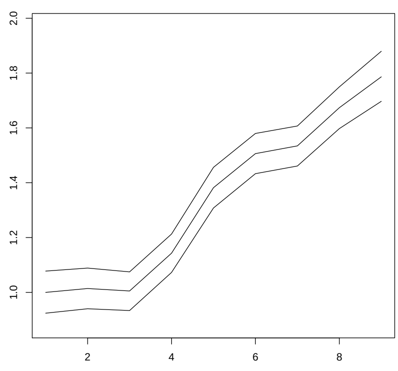
Figure 1: Figures 1a-1b show the parameter estimates for the log-normal distribution that describe the delay distribution. Note that the parameter values are normalised against the mean of the mean parameter on log-scale



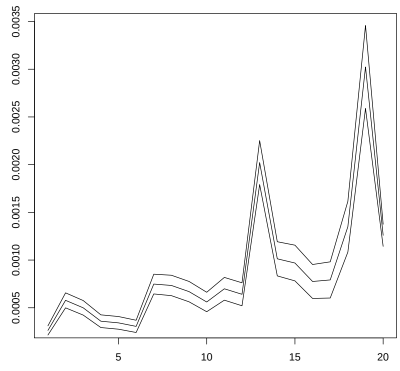
(a) Mean parameter on log-scale (b) Standard deviation parameter on log-scale

Figure 2: Figures 2a-2b show the parameter estimates for the log-normal distribution describing the payment sizes as a function of years since reporting together with 95% confidence intervals.

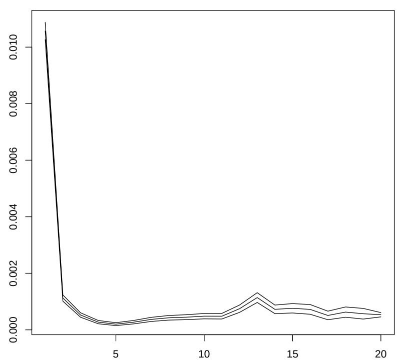
mately 3 years implies that the development process becomes homogeneous making inter-event-times truly exponentially distributed after this point in time.



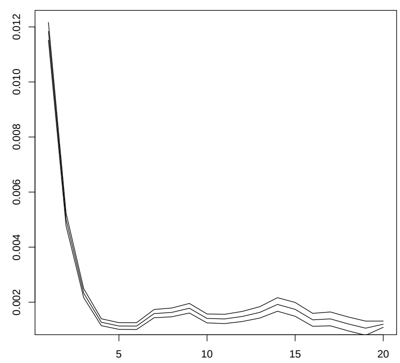
(a) $\hat{\lambda}_i/\hat{\lambda}_1$.



(b) Settled without a payment.



(c) Settled with a payment.



(d) Payment.

Figure 3: Figure 3a displays the ratio of the intensities, $\hat{\lambda}_i/\hat{\lambda}_1$ together with 95% confidence intervals. Figures 3b-3d display the estimated hazard rates together with 95% confidence intervals. The hazard rates are assumed to be constant on 60 days. After 20 periods we assume no further changes in the hazard rates.

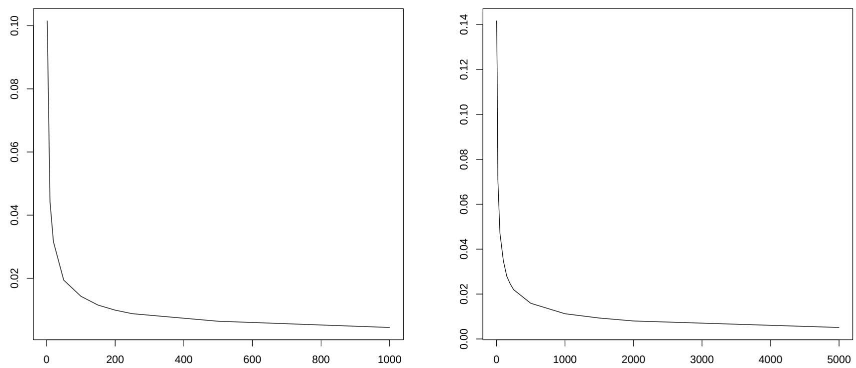
8.5 Determining the number of simulations

An important question when it comes to simulation based reserving is whether or not the simulations have converged or not. Moreover, as implied by the simulation procedures described in Section 5 and 6 it is not surprising that the simulations tend to be computationally heavy. Due to this there is in practice a computational budget w.r.t. time. Consequently it is of interest to analyse how to make the most efficient use of the total number of simulations when split between number of outer, κ, and number of inner, ν, simulations.

In Figure 4 bootstrap error bounds expressed in terms of coefficients of variation for the BE-reserve and the 95% reserve percentile are shown when the number of (outer) simulations, κ, are varied. From Figure 4 it is seen that the bootstrap error bounds decreases rapidly and are negligible for k larger than ca. 5 000, but are only 2-3% when $\kappa \approx 100$.

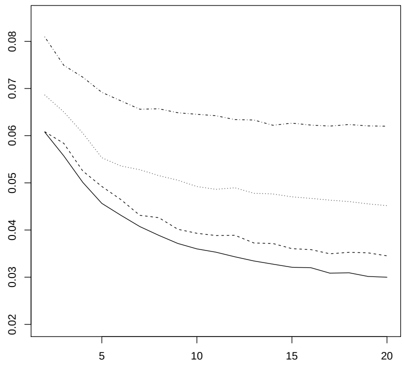
Further, Figure 5 show bootstrap error bounds expressed in terms of coefficients of variation for the reserve risk SCR when varying both the number of outer simulations, κ, as well as the number of inner simulations, ν. Based on Figure 5 it is hence not recommended to use less than $\kappa = 20\ 0000$ outer and $\nu = 10$ inner simulations. It is also worth noting that it is more important to have a large number of outer simulations. Note that this is reasonable, since the number of outer simulations in effect will span the CDR-distribution, i.e. $\mathcal{D}_{\kappa}^{\nu}(t+1)$ from 11, whereas for each of the outer simulations there is only need to simulate enough trajectories in order for the updated (point estimate) BE reserve to become sufficiently accurate.

Based on the above, unless otherwise specified all SCR-calculations are based on $\kappa = 20\ 000$ outer simulations and $\nu = 10$ inner simulations.



(a) (b)

Figure 4: Both panels show bootstrap error bounds expressed as coefficients of variation as a function of number of (outer) simulations. *Panel* (A): Bootstrap error bound for the best estimate reserve as a function of number of (outer) simulations. *Panel* (B): Bootstrap error bound for the 95% percentile of the reserve distribution as a function of number of (outer) simulations.



(a)

Figure 5: Bootstrap error bound expressed as coefficients of variation for $\widehat{SCR}_R(t)$ from (12). Dash-dotted, dotted, dashed and solid lines corresponds to $\kappa = 5\ 000$, 10 000, 20 000 and 30 000 outer simulations, respectively, and inner simulations are displayed on the x-axis (corresponding to ν).

8.6 In-sample performance

Above we have discussed trends in data together with various modelling assumptions. A natural question to address is the in-sample fit produced by the micro model. One way of doing this is to consider the forecasting performance of the model w.r.t. realised payments compared with one-year-ahead forecasted payments. That is, for each occurrence year, based on the information that successively becomes known a one-year-ahead forecast of payments is made. Further, these payments may be categorised as belonging to either RBNS or IBNR claims seen from the start of the year that is being forecasted. Figure 6 show the result of this comparison for a number of occurrence years. From Figure 6 it is evident that that the above model, using piecewise constant intensities together with standard distributions provide a remarkably good fit for both RBNS and IBNR claims.

Figure 7 show a similar evaluation of the chain-ladder method, which also provide a reasonable fit. Since the standard chain-ladder method is distribution free we also make scatter plots comparing the year-on-year development of cumulative payments together with the standardised residuals

$$\hat{e}_{i,k}^{CL} = \frac{c_{i,k+1} - \hat{f}_k c_{i,k}}{\sqrt{c_{i,k}}}, \tag{15}$$

where $\hat{f}_k$ is the standard chain-ladder development factor for projecting payments from development year k to $k+1$, and $c_{i,k}$ corresponds to the observed cumulative payments for claims year i up to development year k. This is in line with [26] and one can note that the particular choice of residuals are in line with the standard variance assumption of the distribution free chain-ladder method. The results of this analysis is shown in Figure 8. From Figure 8 it is seen that the linear year-on-year development pattern seems reasonable, since the development factors are close to the theoretical regression line. Regarding the chain-ladder variance assumption the residuals from Figure 8 may indicate that the standard chain-ladder variance assumption is not suitable. We refrain from further analysis of the suitability of the chain-ladder method for the particular data due to the small number of data points and refer the interested reader to [26] where more test procedures are described in full detail.

Consequently, based on the above there is no further need to analyse the specific building blocks of the model in more detail.

8.7 Out-of-sample performance

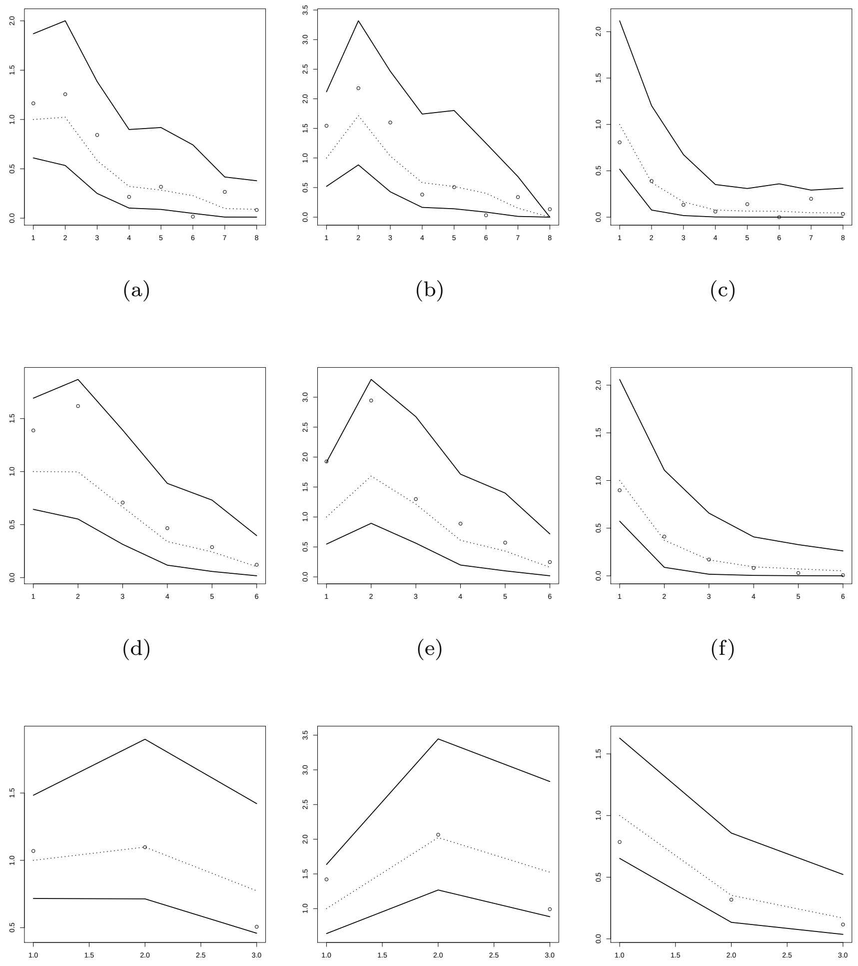
In Section 8.6 it is concluded that the model fit for the micro model seems to be adequate and that the chain-ladder method should provide a reasonable model for comparison. We will now proceed to analyse the out-of-sample performance of the models.

Figure 9 summarises the out-of-sample performance for the micro model w.r.t. performance of the claims reserve when split into RBNS and IBNR claims for all accident years $\leq x + 6$ and $\leq x + 7$, out of a total of 9 accident years (i.e. $x, \ldots, x + 8$). From Figure 9 it is seen that for the best estimate reserve for accident years $\leq x+6$ the performance is good for both RBNS and IBNR reserve behaviour; the actual reserve is contained within the bulk of both the RBNS and IBNR claims reserve distributions, as expected. Moreover, it is seen that the CDR-distribution, i.e. $\mathcal{D}^{\nu}_{\kappa}(t+1)$ from (11), is centred around zero, which is an indication of that the micro model has not introduced any substantial reserve bias.

Further, when inspecting the analogous results for accident years $\leq x+7$ the situation is different. The reserve behaviour is still good for the IBNR claims, but the realised RBNS claims are very small in comparison with the predicted distribution. Moreover, by inspecting the CDR-distribution it is seen that it is still centred around zero, as it should, but that the true outcome corresponds to a huge profit. Based on the CDR-distribution one may also deduce that the IBNR part of the claims reserve is non-negligible. Regarding the extreme outcome of the RBNS reserve, recall that in Section 8.2 it was stated that the claims handling time had been shortened during the last accident years due to changes in the claims handling process. Furthermore, one can note that even though the claims handling times have been shortened, the IBNR reserve still performs well. This type of information may be included into the micro model *a priori* by using expert judgment in a way not possible for claims triangle based macro models.

To summarise the above analysis, the out-of-sample performance of the micro model is behaving as expected.

Turning to the comparison with chain-ladder. In Figure 10 the results for



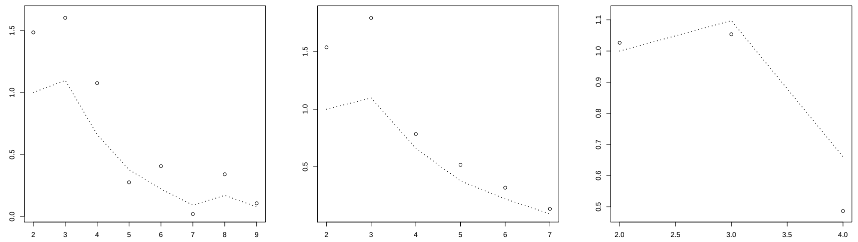
(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Figure 6: *Columns*: Yearly total payments, RBNS payments and IBNR payments, respectively, per development year. *Rows*: Occurrence year x, $x+2$, $x+5$, respectively. In all figures circles corresponds to actual payments, dotted lines corresponds to mean payments for next year according to the micro-model, and solid lines corresponds to 95% confidence intervals. All results are normalised against the mean for development year 2 for each occurrence year.

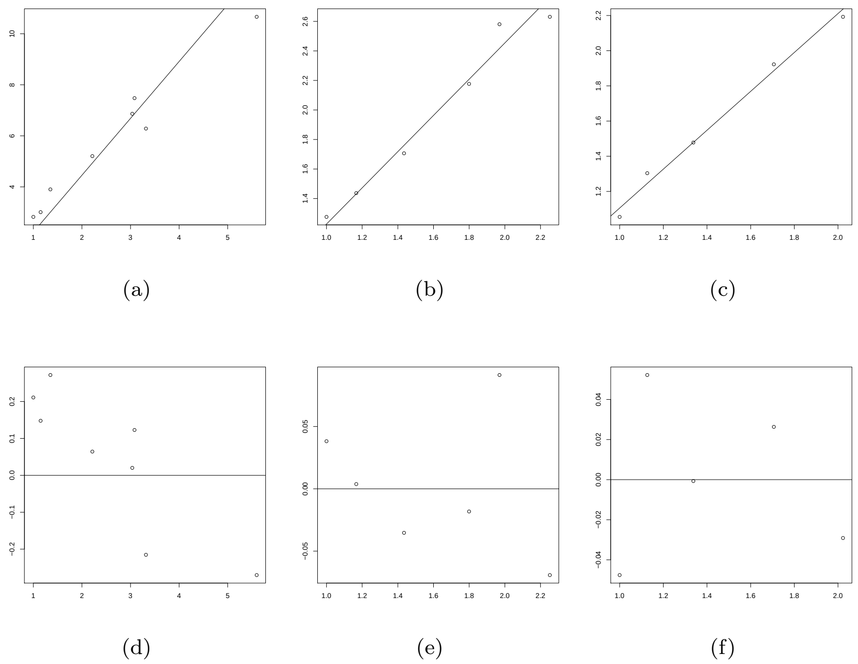
total reserves and payments for both the chain-ladder method and micro model are summarised. All results are normalised with the corresponding total initial micro model reserves. For accident years $\leq x+6$ it is clear that the models are



(a) (b) (c)

Figure 7: Yearly total payments for occurrence year x, $x+2$, $x+5$, respectively, per development year for chain-ladder. In all figures circles corresponds to actual payments, dotted lines corresponds to mean payments for next year according to chain-ladder. All results are normalised against the chain-ladder mean for development year 2 for each occurrence year.

comparable, but the micro model give slightly more conservative loss percentile estimates, being somewhat more skewed towards higher loss percentiles. It can also be added that the total initial micro reserve is slightly higher than the corresponding chain-ladder reserve. Moreover, as opposed to the micro model, the chain-ladder method produces a CDR-distribution which is not centred around zero, which indicates that the method is perhaps not fully aligned with data. This even clearer for accident years $\leq x+7$, but here the CDR-distribution for the chain-ladder method is also considerably wider than the corresponding distribution for the micro model. Further, by analysing the payment distribution and reserve distribution for the chain-ladder method it is seen that both miss the actual outcomes completely. This is, as noted above, a consequence of the change in the claims handling process. One can also note that for the micro model, even though it overestimates the reserve it is clear that the payment distribution still seems to capture the overall dynamics reasonably well. A more detailed analysis of the sub-division between payments coming from IBNR and RBNS claims shows that, in particular, the IBNR payment distribution is working well, see Figure 11. A final remark concerning the reserve risk for accident years $\leq x+7$ is that the initial chain-ladder reserve is approximately 30% greater than the corresponding micro model reserve. In this situation the micro model reserve is believed to be more accurate, since it still captures the IBNR dynamics well. Moreover, when analysing the situation for accident years $\leq x+8$ the micro model produces a decrease in RBNS reserve, due to volume changes in the number of open claims, which results in that the predicted distribution again performs well. This is not the case for the chain-ladder method which continues to overshoot.



(a) (b) (c)

(d) (e) (f)

Figure 8: *First row*: from left to right, scatter plot of normalised cumulative yearly payments $c_{i,k}$ are plotted against $c_{i,k+1}$ together with a line (without intercept) with slope corresponding to the standard chain-ladder development factor $\hat{f}_k$, $k = 1, 3, 4$. *Second row*: from left to right, standardised residuals $\hat{e}_{i,k}$ from (15) are plotted against $c_{i,k}$ for $k = 1, 3, 4$.

8.8 Some comments on the relation between the micro model and standard formula calculations for Solvency II non-life insurance risks

Under the Solvency II insurance regulation most non-life insurance companies will use so-called "standard formula" calculations of reserve- and premium risk. These calculations are based on a (translated) log-normal assumption making the 99.5% loss percentile easily calculated once you have decided on the standard deviation in your loss distribution. For non-life insurance the standard deviations of reserve and premium loss distributions are in the range 10-20%, where the standard deviation is expressed as a fraction of the BE reserve, see [10, Annex II]. Thus, what is called a standard deviation under Solvency II is in fact a coefficient of variation of a CDR-distribution.

For the above implementation of the micro model the coefficient of variation for the one-year reserve risk is $\approx 10\%$. Consequently, if one would allow for a micro model calibration of Undertaking Specific Parameters (USP) the

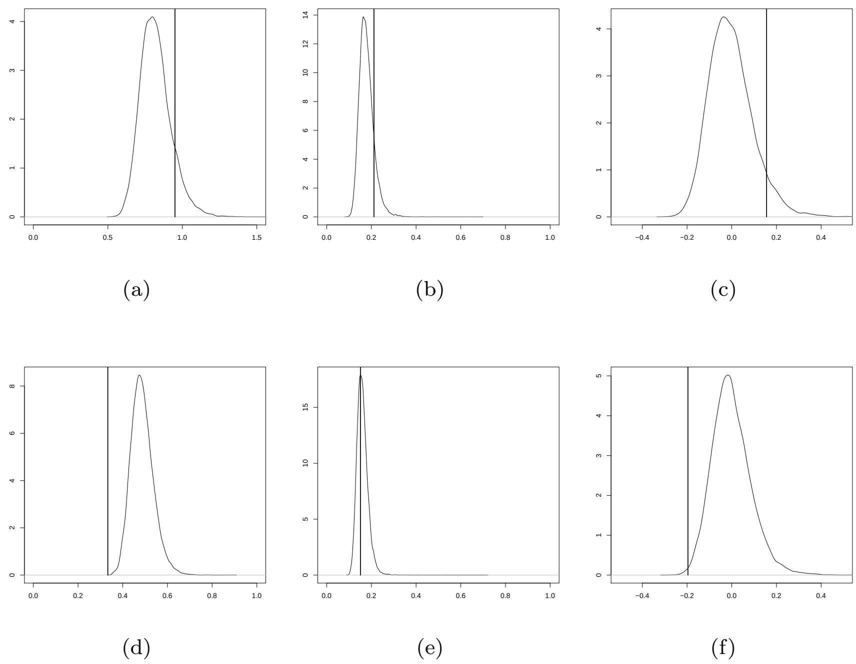


Figure 9: All figures are normalised with the total BE-reserve estimated at time t. *Columns*: RBNS reserve distribution for time $t+1$ estimated at time t, IBNR reserve distribution for time $t+1$ estimated at time t and CDR-distribution, i.e. $\mathcal{D}_\kappa^\nu(t+1)$ from (11), respectively. *Rows*: Accident years $\leq x+6$ and $\leq x+7$, respectively. All results are normalised with the corresponding total BE-reserve estimated at time t. The vertical lines corresponds to true outcomes.

standard deviation (i.e. the coefficient of variation of the CDR-distribution for the claims reserve) obtained is comparable with that of the standard formula capital requirement calculations. Moreover, for more day-to-day fluctuations in the BE-reserve, the micro model is believed to produce considerably less variation than corresponding claims triangle methods, see e.g. the changes in the CDR-distributions for the chain-ladder method from the previous section.

Regarding the premium risk the micro model produces a coefficient of variation of the CDR which is $\approx 5\%$, which is below the corresponding standard formula calibration. These calculations where made using a simple extrapolation of the volume of contracts during the coming year, but we do not, however, see that the effect of shifting the contract volume should have any considerable impact on the coefficient of variation of the premium CDR-distribution. A more important consideration concerning the premium risk calculations is to assess the relevance of the historically fitted model w.r.t. pure out-of-sample forecasts. As an example, recall that in Section 8.2 it was noted that the claims handling

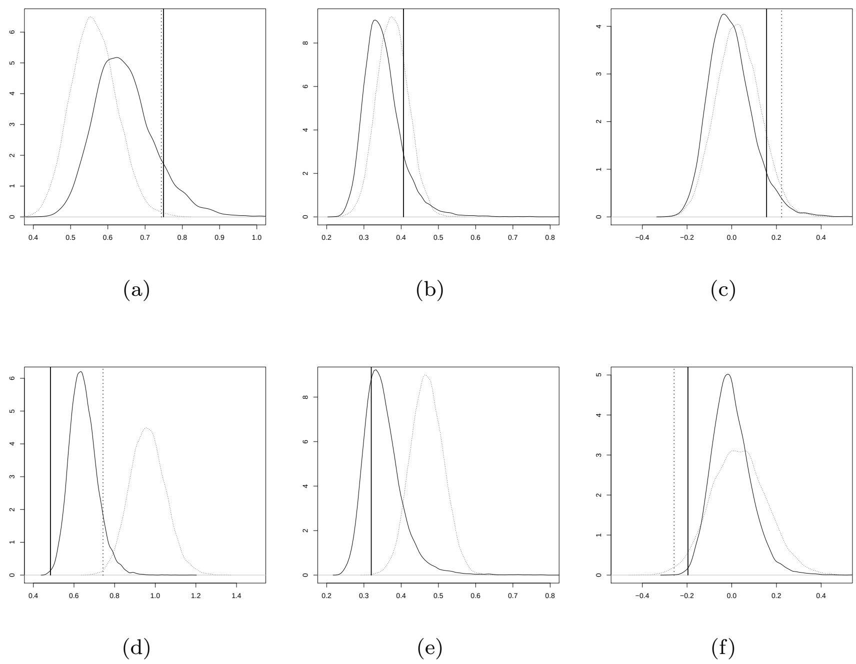


Figure 10: *Columns*: Total reserve distribution for time $t+1$ estimated at time t, total payment distribution for payments in $(t, t+1]$, CDR-distributions, i.e. the distribution analogous to (11), for the chain-ladder method based on the model in Section 7 for accident years $\leq x+6$ and $\leq x+7$, respectively. All results are normalised with the corresponding total *micro* BE-reserve estimated at time t. Solid lines corresponds to micro-model results and dashed lines corresponds to chain-ladder results. The vertical lines corresponds to true outcomes.

times had decreased for the last year, something which was seen in Section 8.7 when assessing out-of-sample performance. Given that you are using a micro model it is however possible to constructively argue for changing certain distributions based on expert judgment.

9 Concluding remarks

The present paper describe how one-year non-life insurance risks according to the Solvency II directive may be consistently calculated for a point process based micro modelling framework as introduced by [32]. This is done continuing the work of [16, 2]. In order for this type of model to be credible to use there is a need for careful model assessment w.r.t. both in-sample and out-of-sample performance, but also regarding robustness due to the number of simulations being

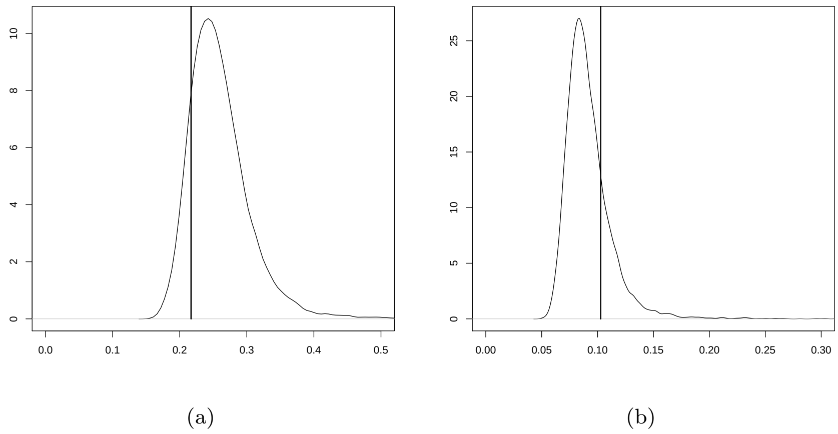


Figure 11: *Columns*: RBNS and IBNR payment distributions from the micro model for payments in $(t, t+1]$ for accident years $\leq x+7$, respectively. All results are normalised with the corresponding total micro BE-reserve estimated at time t. The vertical lines corresponds to true outcomes.

used. All of this has been addressed above. Further, regarding assessment of fit in relation to model complexity more focus was put on actual model performance rather than assessing various model parameters and distributional choices. This is believed to be a reasonable procedure in order to avoid overfitting, since the main objective is to perform out-of-sample predictions.

The main conclusion of the paper is that the micro model approach is feasible to use for one-year Solvency II risk calculations and that the constructive modelling approach will provide additional insights concerning model performance as well as a more flexible modelling framework compared to traditional claims triangle methods. Moreover, the case study in Section 8 suggests that the micro-model's CDR-results are more stable across calendar years compared with the chain-ladder method and that the micro model is better at capturing changes in the claims dynamics – here leading to a large reduction in the BE-reserve, which is believed to be more accurate. Note that in Section 8 there was a decrease in reserves due to that the number of open claims was lowered as a consequence of an improved claims handling process. This could of course have been modelled in more detail by e.g. separating the years before and after the change in claims handling process.

Further, as noted above the presented brute force nested simulation approach is computationally heavy. In Section 8.5 this question was discussed based on how you should use your computational budget and it was found that it is more important to have a large number of outer simulations in the nested simulation procedure. This is reasonable, since the inner simulations are only used to obtain a point estimate of the future BE-reserve. Furthermore, given that one is interested in calculating a consistent "Risk Margin" following [28, 13], i.e. a multi-period cost-of-capital valuation, it is not likely that the above procedure will be computationally possible without carrying out model simplifications or by using bespoke numerical techniques. Moreover, from a practical perspective there is still some skepticism concerning pure simulation based *reserves*, as opposed to a deterministic reserve accompanied with simulation based risk calculations. These comments also suggests the need for future research on analytical (approximate) results concerning fundamental properties of this type of models.

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