Mathematical Statistics
Stockholm University
Research Report **2017:1**,
http://www.math.su.se

Optimal shrinkage-based portfolio selection in high dimensions

February 2017

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Abstract

In this paper we estimate the mean-variance (MV) portfolio in the high-dimensional case using the recent results from the theory of random matrices. We construct a linear shrinkage estimator which is distribution-free and is optimal in the sense of maximizing with probability 1 the asymptotic out-of-sample expected utility, i.e., mean-variance objective function. Its asymptotic properties are investigated when the number of assets p together with the sample size n tend to infinity such that $p/n \to c \in (0, +\infty)$. The results are obtained under weak assumptions imposed on the distribution of the asset returns, namely the existence of the fourth moments is only required. Thereafter we perform numerical and empirical studies where the small- and large-sample behavior of the derived estimator is investigated. The suggested estimator shows significant improvements over the naive diversification and it is robust to the deviations from normality.

JEL Classification: G11, C13, C14, C58, C65

Keywords: expected utility portfolio, large-dimensional asymptotics, covariance matrix estimation, random matrix theory.

1 Introduction

In the seminal paper of Markowitz (1952) the author suggests to determine the optimal composition of a portfolio of financial assets by minimizing the portfolio variance assuming that the

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