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# On Finding Equilibrium Stopping Times for Time-Inconsistent Markovian Problems

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## Abstract

Standard Markovian optimal stopping problems are consistent in the sense that the first entrance time into the stopping set is optimal for each initial state of the process. Clearly, the usual concept of optimality cannot in a straightforward way be applied to non-standard stopping problems without this time-consistent structure. This paper is devoted to the solution of time-inconsistent stopping problems with the reward depending on the initial state using a game-theoretic approach in which each state of the process corresponds to a player in the game. More precisely, we give a precise equilibrium definition – of the type subgame perfect Nash equilibrium based on pure Markov strategies. Such equilibria do not always exist. We, however, develop an iterative approach to finding such equilibrium stopping times for a general class of problems and apply this approach to one-sided stopping problems on the real line. We furthermore prove a verification theorem based on a set of variational inequalities which also allows us to find equilibria. As an application of the developed theory we study a selling strategy problem under exponential utility and endogenous habit formation.

*Keywords:* Markov process, Optimal stopping, Subgame perfect Nash equilibrium, Time-inconsistency, Variational inequalities.

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