



Tangency portfolio weights for singular covariance matrix in small and large dimensions: estimation and test theory

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TARAS BODNAR^a, STEPAN MAZUR^{b,1}, KRZYSZTOF PODGÓRSKI^c, JOANNA TYRCHA^a

^a *Department of Mathematics, Stockholm University, SE-10691 Stockholm, Sweden*

^b *Department of Statistics, Örebro University School of Business, SE-70182 Örebro, Sweden*

^c *Department of Statistics, Lund University, SE-22007 Lund, Sweden*

Abstract

In this paper we derive the finite-sample distribution of the estimated weights of the tangency portfolio when both the population and the sample covariance matrices are singular. These results are used in the derivation of a statistical test on the weights of the tangency portfolio where the distribution of the test statistic is obtained under both the null and the alternative hypotheses. Moreover, we establish the high-dimensional asymptotic distribution of the estimated weights of the tangency portfolio when both the portfolio dimension and the sample size increase to infinity. The theoretical findings are implemented in an empirical application dealing with the returns on the stocks included into the S&P 500 index.

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^{1*} Corresponding author. E-mail address: stepan.mazur@oru.se. The authors appreciate the financial support of Swedish Research Council: Dynamical stochastic models for efficient spatial analysis of linkages in financial markets, Statistics in the Empirical Sciences.