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SEIRS epidemics in growing populations

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Abstract

An SEIRS epidemic with disease fatalities is introduced in a growing population (modelled as a super-critical linear birth and death process). The study of the initial phase of the epidemic is stochastic, while the analysis of the major outbreaks is deterministic. Depending on the values of the parameters, the following scenarios are possible. i) The disease dies out quickly, only infecting few; ii) the epidemic takes off, the *number* of infected individuals grows exponentially, but the *fraction* of infected individuals remains negligible; iii) the epidemic takes off, the *number* of infected grows initially quicker than the population, the disease fatalities diminish the growth rate of the population, but it remains super critical, and the *fraction* of infected go to an endemic equilibrium; iv) the epidemic takes off, the *number* of infected individuals grows initially quicker than the population, the diseases fatalities turn the exponential growth of the population to an exponential decay.

Keywords: SEIRS epidemic; threshold quantities; initial growth; endemic level.

1 Introduction

Infectious diseases remain a threat for developing countries as well as for developed countries. Many mathematicians focus their efforts to understand the dynamics of infectious diseases, in order to find the conditions to eradicate them. In mathematical modelling of infectious disease epidemics, the population in which the disease is spreading is partitioned in several compartments according to the status of the individuals, related to the disease. Every epidemic model has at least, the compartment I of the infectious individuals who are infected and able to transmit the disease to others through contact, and the compartment S of the susceptible individuals (those who are not infected but may be infected if they contact an infectious individual). Two other compartments often used are the compartment E of the exposed or latent individuals who are already infected but not yet able to transmit the disease to others, and the compartment R of the recovered or removed individuals (those who are healed from the disease with a permanent or non-permanent immunity). In a $SEIR$ epidemic, a susceptible individual infected through a contact with an infectious, becomes infected and latent; at the end of the latent period he/she becomes infectious and at the end of the infectious period he/she recovers

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