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Finite Sample Size Bounds on the Variance Estimator in Non-Gaussian General Linear Models

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Abstract

We consider bounds on the variance of the standard unbiased variance estimator in a general non-Gaussian linear model for finite sample sizes. In particular we obtain bounds that are sharp in the sense that both the lower and upper bound will converge to the same asymptotic limit when scaled with the sample size. Further, these bounds are independent of covariate information. Due to this we may also obtain unconditional variance bounds for the situation with random covariates. The above results rely on a result in Atiqullah (1962) which is stated without proof. We provide a proof of this result, both for easy reference, but also since the derivation of the variance bounds rely on an observation in the construction of this proof.

Keywords: General linear models, non-Gaussian error terms, moments of variance estimators, finite sample size bounds, random covariates, unconditional bounds

1 Introduction

In the present note we will consider the variance of the standard unbiased variance estimator in a general linear model (GLM) with non-Gaussian error terms. For this class of models the covariance of the generalized least squares estimator $\hat{\boldsymbol{\beta}}$ is well-known, but an explicit expression for the variance of $\hat{\sigma}^2$ is not as widely known. In [2] an expression for this variance is provided without proof for a mixed GLM with non-Gaussian homoscedastic error terms. This result allows us to obtain finite sample size bounds on the variance of $\hat{\sigma}^2$ which are independent of the covariates in the non-Gaussian GLM. Moreover, the bounds which we obtain are sharp in the sense that