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Closed-Form Estimator for the Matrix-Variate Gamma Distribution

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Abstract

In this paper we present a novel closed-form estimator for the parameters of the matrix-variate gamma distribution. The estimator relies on the moments of a transformation of the observed matrices, and is compared to the maximum likelihood estimator (MLE) through a simulation study. The study reveals that the suggested estimator outperforms the MLE, in terms of estimation error, when the underlying scale matrix parameter is ill-conditioned or when the shape parameter is close to its lower bound. In addition, since the suggested estimator is closed-form, it does not require numerical optimization as the MLE does, thus needing shorter computation time and is furthermore not subject to start value sensitivity or convergence issues. Finally, using the proposed estimator as start value in the optimization procedure of the MLE is shown to substantially reduce computation time, in comparison to using arbitrary start values.

1 Introduction

The matrix-variate gamma distribution is a generalization of the univariate gamma distribution to the set of positive-definite and symmetric matrices. It is also a more general form of the classical Wishart distribution and a popular approach to model e.g. the stochastic properties of covariance matrices of financial asset returns, which in turn has numerous important applications. For an overview of the matrix-variate gamma distribution and some of its properties, see Gupta and Nagar (2000).

We denote a symmetric and positive definite $p \times p$ matrix $\mathbf{A}$ that follows a matrix-variate gamma distribution with shape parameter α and symmetric scale matrix $\mathbf{\Sigma}$ as $\mathbf{A} \sim MG_p(\alpha, \mathbf{\Sigma})$, where $\alpha > (p-1)/2$, and $\mathbf{\Sigma} > 0$. Let $\mathbf{A}_1, \dots, \mathbf{A}_n$ be a sample of i.i.d. observations of the matrix-variate gamma distribution, and let $\overline{\mathbf{A}}$ be its sample mean. In general, the maximum likelihood method is the most efficient way of estimating the parameters α and $\mathbf{\Sigma}$, given such a sample. The maximum likelihood estimates of these parameters is obtained by solving the following system of equations for α and $\mathbf{\Sigma}$:

$$\psi_p(\alpha) = \frac{\sum_{k=1}^{n} \ln\left(|\mathbf{A}_k|\right)}{n} - \ln\left(|\mathbf{\Sigma}\right)| \tag{1}$$

$$\mathbf{\Sigma} = \frac{\overline{\mathbf{A}}}{\alpha}, \tag{2}$$

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