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Competing frogs on $\mathbb{Z}^d$

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Abstract

A two-type version of the frog model on $\mathbb{Z}^d$ is formulated, where active type i particles move according to lazy random walks with probability p_i of jumping in each time step ($i = 1, 2$). Each site is independently assigned a random number of particles. At time 0, the particles at the origin are activated and assigned type 1 and the particles at one other site are activated and assigned type 2, while all other particles are sleeping. When an active type i particle moves to a new site, any sleeping particles there are activated and assigned type i, with an arbitrary tie-breaker deciding the type if the site is hit by particles of both types in the same time step. We show that the event G_i that type i activates infinitely many particles has positive probability for all $p_1, p_2 \in (0, 1]$ ($i = 1, 2$). Furthermore, if $p_1 = p_2$, then the types can coexist in the sense that $\mathbb{P}(G_1 \cap G_2) > 0$. We also formulate several open problems. For instance, we conjecture that, when the initial number of particles per site has a heavy tail, the types can coexist also when $p_1 \neq p_2$.

Keywords: Frog model, random walk, competing growth, coexistence.

AMS 2010 Subject Classification: 60K35.

1 Introduction

The so called frog model on $\mathbb{Z}^d$ is driven by moving particles on the sites of the $\mathbb{Z}^d$-lattice. Each site $x \in \mathbb{Z}^d$ is assigned an initial number $\eta(x)$ of particles, where $\{\eta(x)\}_{x\in\mathbb{Z}^d}$ are independent and identically distributed. We write ν for the product measure defined by this initial particle distribution. Each particle is then independently equipped with a discrete time simple symmetric random walk, denoted for particle $j = 1, \ldots, \eta(x)$ at the site x by $(S_n^{x,j})_{n\in\mathbb{N}}$ and encoded by jumps rather than sites. A particle starts moving from its initial location and the associated random walk then specifies the movement of the particle in each time step. The set of all these random walks is denoted by $S = \{(S_n^{x,j})_{n\in\mathbb{N}} : x \in \mathbb{Z}^d, j = 1, \ldots, \eta(x)\}$. At time 0, the particles at the origin are activated, while all other particles are sleeping. When a particle is activated, it starts moving according to its associated random walk so that, in each time step, it moves to a uniformly chosen neighboring site. When a site is visited by an active particle, any sleeping particles at the site are activated and start moving. If the origin is non-empty, this means that the set of activated particles grows to infinity.

The model has previously been studied e.g. with respect to transience/recurrence [25], the shape of the set of visited sites [2, 3] and extinction/survival for a version of the model including death of active particles [4]. Here we introduce a two-type version of the model, where an active particle can be of either of two types. We study the possibility for the types to activate infinitely many particles and investigate in particular the event of coexistence, which is said to occur if both types activate infinitely many particles. Similar questions have been studied for other competition models on $\mathbb{Z}^d$, for instance the so-called Richardson model where a site becomes