Mathematical Statistics
Stockholm University
Research Report **2019:5**,
http://www.math.su.se

Exact long time behavior of some regime switching stochastic processes

Filip Lindskog and Abhishek Pal Majumder

April 2019

Abstract

Regime switching processes have proved to be indispensable in the modeling of various phenomena, allowing model parameters that traditionally were considered to be constant to fluctuate in a Markovian manner in line with empirical findings. We study diffusion processes of Ornstein-Uhlenbeck type where the drift and diffusion coefficients a and b are functions of a Markov process with a stationary distribution π on a countable state space. Exact long time behavior is determined for the three regimes corresponding to the expected drift: $E_\pi a(\cdot) > 0, = 0, < 0$, respectively. Alongside we provide exact time limit results for integrals of form $\int_0^t b^2(X_s)e^{-2\int_s^t a(X_r)dr}ds$ for the three different regimes. Finally, we demonstrate natural applications of the findings in terms of Cox-Ingersoll-Ross diffusion and deterministic SIS epidemic models in Markovian environments. Exact long time behaviors are naturally expressed in terms of solutions to the well-studied fixed-point equation in law $X \overset{d}{=} AX + B$ with $X \perp\!\!\!\perp (A, B)$.

1 Introduction

Models based on regime switching stochastic processes have received considerable attention for their applications in quantitative finance, actuarial science, economics, biology and ecology. In quantitative finance, volatility, interest rates and asset prices are subjects to risky market environments that fluctuate over different regimes in a Markovian manner. Understanding how critical parameters (that determine stability or instability of the process of interest) characterizing the "switching regimes" vary stochastically over time and affect the long time behavior of the overall process is essential for making short and long term predictions. Examples of such applications are [3], [7], [19] and [21] in the context of stochastic volatility modelling in financial market; [44] considering stochastic interest rate models with Markov switching; [27], [31] and [40] studying long term behavior of stock returns and bond pricing. Similar to quantitative finance, regime switching stochastic processes are frequently used in actuarial science for solvency investigations, e.g. [1], mortality modeling, e.g. [20], and in the context of disability insurance, e.g. [14].

Monographs containing both the theoretical foundations and applications of regime switching processes are [32] and [43]. Significant contributions to the theoretical foundation are [37], [38] and [39] by Jinghai Shao. A common theme of these works is a stochastic dynamical system $(Y_t, X_t)_{t\geq 0}$, where the process of interest $Y := (Y_t)_{t\geq 0}$ is affected by the process $X := (X_t)_{t\geq 0}$ that describes the dynamics of a switching environment. For a class of general diffusion processes Y the aforementioned works investigated necessary and sufficient conditions under which properties related to stability/instability such as geometric/polynomial ergodicity [37],[38], positive/null recurrence or transience [39], explosivity, existence and uniqueness of moments of stationary distributions hold. In a similar context [6] (and references therein) addresses questions related with survival or extinction of competing species in Lotka-Volterra model influenced by switching parameters in terms of the underlying hidden Markov environment. A main theme is the analysis of persistence (see section 4 or Theorem 4.1 of [6]) phrased in terms of so-called Lyapunov drift type criteria and similar concepts. In [12] a large class of general regime switching Markov processes are considered where a type of condition referred to as "geometric contractivity" ensures exponential stability of the overall process. In contrast to the general stability results described above, there are very few works giving exact characterizations of long time behaviors, which are inevitably model specific. In this paper, we