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The Schelling model on $\mathbb{Z}$

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Abstract

A version of the Schelling model on $\mathbb{Z}$ is defined, where two types of agents are allocated on the sites. An agent prefers to be surrounded by other agents of its own type, and may choose to move if this is not the case. It then sends a request to an agent of opposite type chosen according to some given moving distribution and, if the move is beneficial for both agents, they swap location. We show that certain choices in the dynamics are crucial for the properties of the model. In particular, the model exhibits different asymptotic behavior depending on whether the moving distribution has bounded or unbounded support. Furthermore, the behavior changes if the agents are lazy in the sense that they only swap location if this strictly improves their situation. Generalizations to a version that includes multiple types are discussed. The work provides a rigorous analysis of so called Kawasaki dynamics on an infinite structure with local interactions.

1 Introduction

The Schelling model of segregation was formulated by Thomas Schelling in the 1960's as an attempt to explain the occurrence of racial segregation in terms of individual preferences rather than policies of central authorities; see [13, 14]. It is one of the first examples of so-called agent based modeling in the economic literature, where the dynamics is formulated in terms of actions of autonomous agents and the interest then concerns large scale properties of such populations. In probability, this type of models is studied in the area of interacting particle systems. In the work of Schelling, two types are distributed on a finite part of $\mathbb{Z}^d$ $(d = 1, 2)$ that also contains a certain percentage of empty sites and, depending on the number of agents of opposite type in some neighborhood around it, an agent can choose to move to another (empty) location where the neighborhood contains fewer agents of the opposite type. Specifically, if the fraction of agents of opposite type in the neighborhood exceeds a certain threshold, then the agent is dissatisfied and therefore prone to move. One of the main lessons learnt from the model is that already a mild preference for being surrounded by alike agents can lead to massive segregation on a macroscopic level.

The Schelling model has been widely studied in the economic literature, mainly by aid of simulations, but so far there is not much rigorous work. A number of choices have to be made to specify the precise dynamics of the model, e.g. the proportions of the types and empty sites, the choice of the new location of a moving agent and whether moves that do not strictly improve the neighborhood composition for an agent are permitted. We study a version of the model

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