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Sampling Distributions of Optimal Portfolio Weights and Characteristics in Low and Large Dimensions

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Abstract

Optimal portfolio selection problems are determined by the (unknown) parameters of the data generating process. If an investor want to realise the position suggested by the optimal portfolios he/she needs to estimate the unknown parameters and to account the parameter uncertainty into the decision process. Most often, the parameters of interest are the population mean vector and the population covariance matrix of the asset return distribution. In this paper we characterise the exact sampling distribution of the estimated optimal portfolio weights and their characteristics by deriving their sampling distribution which is present in terms of a stochastic representation. This approach possesses several advantages, like (i) it determines the sampling distribution of the estimated optimal portfolio weights by expressions which could be used to draw samples from this distribution efficiently; (ii) the application of the derived stochastic representation provides an easy way to obtain the asymptotic approximation of the sampling distribution. The later property is used to show that the high-dimensional asymptotic distribution of optimal portfolio weights is a multivariate normal and to determine its parameters. Moreover, a consistent estimator of optimal portfolio weights and their characteristics is derived under the high-dimensional settings. Via an extensive simulation study, we investigate the finite-sample performance of the derived asymptotic approximation and study its robustness to the violation of the model assumptions used in the derivation of the theoretical results.

Keywords: sampling distribution; optimal portfolio; parameter uncertainty; stochastic representation; high-dimensional asymptotics