Stockholms universitet

Parameter setting for yield curve extrapolation and the implications for hedging

Vadim Ovtchinnikov

Masteruppsats i matematisk statistik
Master Thesis in Mathematical Statistics

Masteruppsats 2015:2
Försäkringsmatematik
April 2015

www.math.su.se

Matematisk statistik
Matematiska institutionen
Stockholms universitet
106 91 Stockholm

Mathematical Statistics
Stockholm University
Master Thesis **2015:2**
http://www.math.su.se

Parameter setting for yield curve extrapolation and the implications for hedging

Vadim Ovtchinnikov*

April 2015

Abstract

The European Insurance and Occupational Pension Authority (EIOPA) has, in their Solvency II-framework adapted the Ultimate Forward Rate (UFR) methodology to value pension and insurance liabilities. When deriving the interest rate curve the actual market rates until a predefined Last Liquid Point (LLP) are used.Beyond this point the market is assumed to benon-liquid and a poor proxy for liability valuation. EIOPA's solution for valuing long-tailed business is to use the Smith-Wilson extrapolation towards the UFR, and synthetically deriving the rest of the interest rate curve beyond the LLP. Smith-Wilson extrapolation takes no market information into account after the LLP and has a peculiar interest rate sensitivityfor shorter choice of LLP parameters. Under the Solvency II-framework, the regulator aims to build a bridge between asset and liability side of an insurance companies' balance sheets, but based on results demonstrated in this thesisthe asset managers will face practical difficulties when hedging companies' liabilities against yield curve movements. It has been disclosed that incorrect parameter setting will increase the interest rate sensitivity and force companies to become more leveraged. Finally, a sub-optimal hedge portfolio framework has been proposed which constrains the asset allocation to long positions in fixed interest rate and provide companies with a practical solution of how to mitigate their interest rate exposure under current regulation.

*Postal address: Mathematical Statistics, Stockholm University, SE-106 91, Sweden. E-mail: vadim.ovtchinnikov@cavendi.se. Supervisor: Andreas Nordvall Lagerås.

PARAMETER SETTING FOR YIELD CURVE EXTRAPOLATION AND THE IMPLICATIONS FOR HEDGING

Vadim Ovtchinnikov

Abstract
The European Insurance and Occupational Pension Authority (EIOPA) has, in their Solvency II-framework, adapted the Ultimate Forward Rate (*UFR*) methodology to value pension and insurance liabilities. When deriving the interest rate curve the actual market rates until a predefined Last Liquid Point (*LLP*) are used. Beyond this point the market is assumed to be non-liquid and a poor proxy for liability valuation. EIOPA's solution for valuing long-tailed business is to use the Smith-Wilson extrapolation towards the *UFR*, and synthetically deriving the rest of the interest rate curve beyond the *LLP*. Smith-Wilson extrapolation takes no market information into account after the *LLP* and has a peculiar interest rate sensitivity for shorter choice of *LLP* parameters. Under the Solvency II-framework, the regulator aims to build a bridge between asset and liability side of an insurance companies' balance sheets, but based on results demonstrated in this thesis the asset managers will face practical difficulties when hedging companies' liabilities against yield curve movements. It has been disclosed that incorrect parameter setting will increase the interest rate sensitivity and force companies to become more leveraged. Finally, a sub-optimal hedge portfolio framework has been proposed which constrains the asset allocation to long positions in fixed interest rate and provides companies with a practical solution of how to mitigate their interest rate exposure under current regulation.

Keywords
Solvency II — Life-insurance – Pension – Ultimate forward rate methodology — Interest rate sensitivity – Hedge – Last liquid point – Conversion period

Correspondence: vadimo@kth.se

Contents

Introduction

Pension and life-insurance agreements are usually long-tailed. A policyholder enters an insurance contract by paying a premium. The company is in turn obliged to pay out a benefit in case of a future and not yet known insured event. The best estimate of these benefits (expected value) is the guaranteed amounts times the probability of the events happening at every relevant period ahead in time, discounted to today's market value. The probability may be expressed as a function of multiple parameters, where the most significant risks, from the company's perspective, are market risk, mortality risk, longevity risk, lapse risk, operational risk. Endowment and term insurances can stretch to nearly a century and so does the expected liability of the company. The market risk is typically the largest risk on a company's balance sheet and the interest rate sensitivity is often the driving factor. The focus of this thesis will be company's mitigation of the interest rate exposure under EIOPA's regulation and the implications of erroneously set parameters.

Particularities in the interest rate sensitivity were described in Rebel's article from 2012 [1] for EUR denominated liabilities and, in Section 1.2.2, the reader can see that they become even larger for the parameter setting of the SEK denominated liabilities. Furthermore, from the analysis of Section 2.2.2, the reader can see that the choice of the *LLP* parameter has significant practical consequences when setting up a hedge portfolio. The obvious follow-up questions is how the *LLP* parameter has been chosen: Is the methodology chosen by EIOPA when performing the Deep, Liquid and Transparent (DLT) analysis, that determined the *LLP*, really the best approach when considering the whole picture? Has the regulator been too strict by pushing down the *LLP* to 10 years for the Swedish market? An independent analysis of the Swedish swap market has been carried out in Section 1.1. In this thesis, it has been shown that a short *LLP* parameter will increase the interest rate sensitivity and force companies to become more leveraged when mitigating the interest rate exposure, which might lead to increased risk and asset management costs.

1. Methods

The Swedish insurance industry has, throughout the recent years, questioned EIOPA's methodology for liability valuation. After it has become clear that the ultimate forward rate methodology is here to stay, the industry has shifted its focus towards the proposed extrapolation parameters. The main concern has been the difficulty to set up a hedge portfolio in order to match the companies' liabilities. In this thesis, an independent investigation is performed on the choice of the *LLP* based on historical market data. Thereafter, a hedge portfolio has been set up and its performance has been evaluated over time for different choice of *LLP*. Moreover, the performance of the hedge portfolio have also been evaluated for different choices of convergence point (*CP*).

1.1 Analysis of the Swedish swap market

The Swedish insurance industry has recently gained interest for swaps denominated in SEK, since EIOPA's Solvency II-directive considers the swap[1] market to be the best proxy for the derivation of the discounting curve for valuation of insurance liabilities. The discounting curve will be determined based on the market rates, for the tenors for which the market is considered to be active, and thereafter with help of the extrapolation techniques towards a predetermined ultimate forward rate, to which the rates are assumed to converge at the CP.

The 10 year tenor is, according to EIOPA, the *LLP* for the Swedish market. Although paper [3] from Finansinspektionen (the Swedish Financial Supervisory Authority) raises the question if any other points, for example 12 years, 15 years or even 20 years may also be considered as active.

According to Finansinspektionen's assessment, activity in the markets is primarily based on observations from reliable, structured and transparent pricing sources, where quotations can be obtained daily, and where there is also a long price history.

This is in fact a weaker definition than the one EIOPA has used for choosing the *LLP*, where besides being active this point should also be liquid. Liquidity is, in turn, an abstract concept that essentially measures to what extent supply meets demand. There are three principal dimensions to measure liquidity[2]:

- Depth: The amount of trade volume that can be executed without impacting price.
- Tightness (also known as Breadth): The ease of purchase/sale, or the ratio of supply/demand, typically measured by the bid – ask spread.
- Resilience: The amount of time before prices return to pre-large trade levels.

A market is assumed liquid if transactions involving a large quantity of financial instruments can take place without significantly affecting the price of the instruments. Conversely, a market is liquid if financial instruments can readily become converted to cash through an act of buying or selling without causing a significant movement in the price.

The remaining part of this section aims to present an independent investigation of which swap term may be considered as a feasible candidate for the *LLP* on the Swedish swap market. The main focus will be on the depth and tightness criteria. The resilience criteria is the speed with which prices return to former levels after a large transaction. High frequency data (bid – ask rates and volumes) would be needed in order to carry out this analysis. Due to lack of data the third liquidity criterion have been left out for future studies.

1.1.1 Analysis of the tick count

Swaps are over the counter (OTC) traded financial instruments and, therefore, the information on volume is less accessible compared to financial instruments that are being traded on an exchange. A good proxy for volume is the tick count from Bloomberg's platform. In Figure 1, you find the monthly aggregated tick count from August 2013 until January 2014 for SEK denominated swaps. By observing the scale of the two figures, it becomes clear that the number of ticks registered in Bloomberg's platform is substantially larger for EUR. Overall, 18.1 million ticks were registered in total for EUR, which can be compared with only 1.7 million ticks for SEK during this period. This pattern can be compared with Figure 2 that illustrates the tick count for EUR denominated swaps for the same period. Please notice that the scale on the tick count is logarithmic in both figures.

In SEK's case, when observing the average tick count for this particular sample the 4 to 20 years tenors seem to be traded on approximately the same level. The 30 years tenor seems to be less traded. For the 12 to 15 years points a slight

[1] For further study of the interest rate swap mechanics the chapter starting on page 147 in [2] is advised

[2] Page 49 in [4]

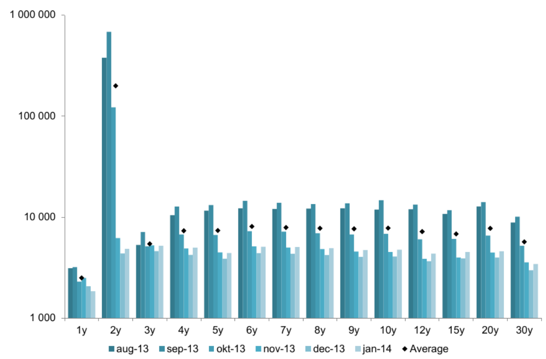

Figure 1. Bloomberg's tick count (SEK)

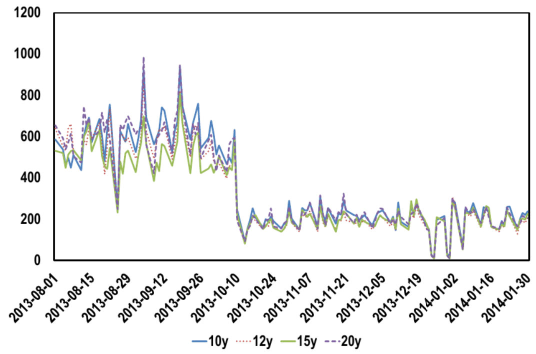

Figure 3. Bloomberg's tick count (SEK)

undershoot is observed compared to, the tick count observed for, 10 and 20 years tenors. It is evident that the 30 year tenor is less traded than the nearest points left of it.

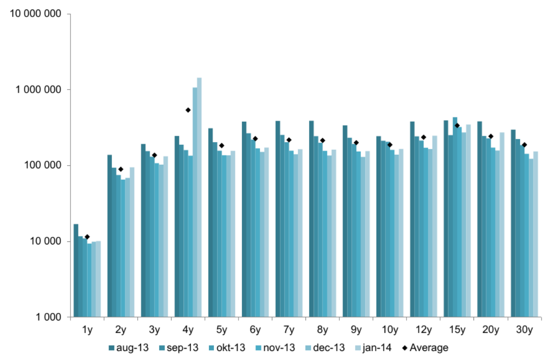

Figure 2. Bloomberg's tick count (EUR)

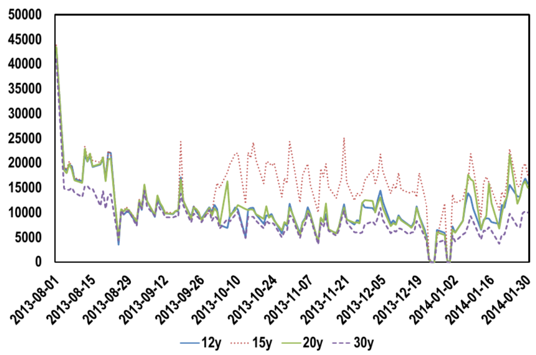

Figure 4. Bloomberg's tick count (EUR)

In EUR's case an undershoot in the average tick count for the 30 year (187 466) can be observed, which is approximately on the same level as the 10 year (187 631) tenor.

In Figure 3 the tick count of SEK denominated swaps for tenors 10, 12, 15, and 20 years can be observed. Moreover, in the following Figure 4 the tick count of EUR denominated swaps for terms 12, 15, 20, and 30 years can be observed as a time series. For SEK's case, the tick count for all four tenor are correlated and are approximately at the same level throughout the time series.

1.1.2 Analysis of bid–ask spread

In this study, EIOPA's second criterion has been examined, the tightness, by observing the weekly bid–ask spread deduced from Bloomberg data from 4 January 2008 until 31 January 2014. The average bid–ask spread and its standard deviation for swaps denominated in SEK are depicted in Figure 5. From the graph it can be observed that the average spreads are about 3 basis points (bp)—hundredths of a percentage point— for all the tenors up to 15 years, after which the average nearly doubles to 5.90 bp for 20 year tenors. Furthermore, one can observe that the standard deviation for 5, 9, and 10 years tenors are slightly narrower than their neighbours and the average standard deviation, for these three points, is 0.97 bp. Moreover, the average standard deviation for the tenors 3 to 8 years is on average 1.17 bp. It is increasing to 1.85 bp for the 10 year tenors and drops slightly to 1.51 bp for the 15 year terms. For 20 year tenors, it shoots up to 2.73 bp. From this sample, the tightness is slightly reduced above the 10 year point, but a significant increase in spread is observed for the 20 year tenor.

Table 1 illustrated the average and standard deviations of bid–ask spread of SEK denominated swaps, per tenor, at the right end of the tail. To have a reference point, the SEK denominated swaps data can be compared with the EUR denominated swap data provided in Figure 6 and Table 2. Both graphs are plotted using the same scale. In the tables, $\bar{x}$ and σ denote average and standard deviation.

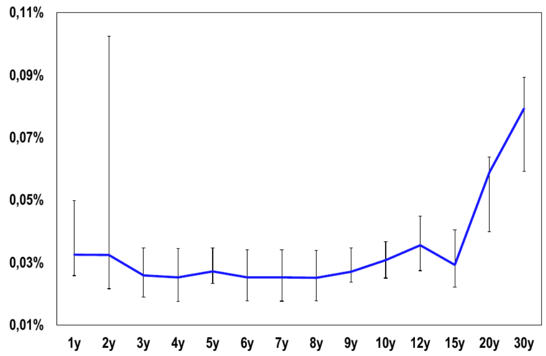

Figure 5. Bloomberg's bid–ask spread (SEK): average and standard deviation (Error)

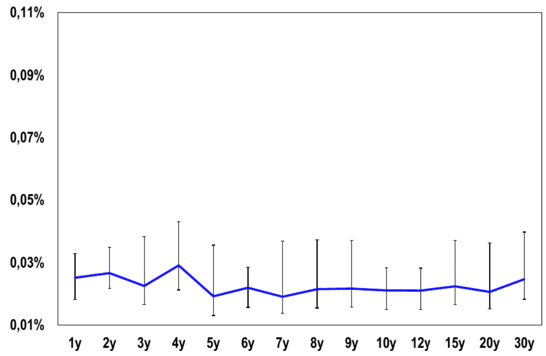

Figure 6. Bloomberg's bid–ask spread (EUR): average and standard deviation (Error)

Table 1. Bid-Ask spread (SEK)
Tenor (year)

basis points	10y	12y	15y	20y	30y
$\bar{x}$	3.08	3.56	2.93	5.90	7.94
σ ($\forall x_i$)	1.00	1.85	1.51	2.73	2.35
σ ($x_i \geq \bar{x}$)	0.59	0.93	1.12	0.49	1.00
σ ($x_i < \bar{x}$)	0.57	0.81	0.71	1.91	2.01

1.2 The hedge portfolio

In this study an analysis was conducted. Two large Swedish life-insurance companies have made a contribution to this work by sharing their liability cash flows, the best estimates of future guaranteed pay-out patterns, with monthly periodicity. Since the companies want to be incognito they are denoted as Company A and B throughout this paper. Their cash flows have been normalized so that the pay-out at month one is 1,000 SEK, and can be observed in Figure 7. Notice that Company A have a smoother pay-out pattern with a shorter duration than Company B.

1.2.1 Liability valuation

The liability valuation is performed in accordance with the Smith-Wilson methodology described in Annex of Subsection 5.B of [5]. With the *UFR* parameter $\omega = \log(1+4.2\%)$[3] and the tolerance parameter $\tau = 1\text{bp}$[4]. The *LLP* parameter for

[3] Page 83 section 4. in [5]
[4] Page 87 section 15.A. (11.3) in [5]

Table 2. Bid–ask spread (EUR)
Tenor (year)

basis points	10y	12y	15y	20y	30 y
$\bar{x}$	2.11	2.11	2.25	2.07	2.48
σ ($\forall x_i$)	1.27	1.22	1.51	1.43	1.61
σ ($x_i \geq \bar{x}$)	0.73	0.72	1.47	1.56	1.51
σ ($x_i < \bar{x}$)	0.61	0.60	0.59	0.54	0.65

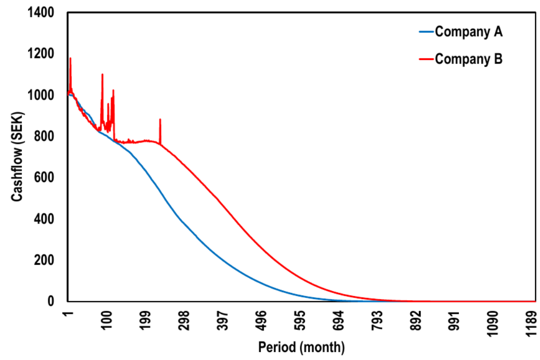

Figure 7. Normed liability cash flows (SEK)

all EEA currencies (including SEK) are chosen according to the results of the Deep, Liquid and Transparent (DLT) criteria assessment[5]. The results of which are provided in Table 2[6] where it is stated that the Swedish risk-free rate should be derived from annual interest rate swaps[7] based on the market points up to the 10 years tenor. Conclusively, the *LLP* parameter should be set to 10 years tenor according to the regulation. Furthermore, the convergence point[8] (*CP*) is set to 60 years for SEK denominated liabilities. This is an increase from the previous indications of 20 or 50 years which were discussed in the LTGA document [4].

The Credit risk adjustment[9] has been omitted in the valuation of the liabilities, but since it was done consistently for all the periods in the projection, it will not affect the conclusions that were made in this study.

[5] Page 38 section 6.B. 134 in [5]
[6] Page 28 section 4.B. in [5]
[7] Page 27 section 4.A. in [5]
[8] Page 39 section 6.E. in [5]
[9] Section 5.B in [5]

1.2.2 Liability's interest rates sensitivity

The interest rate sensitivity can be measured by observing the dollar value of a basis point (DV01) defined as:

$$\text{DV01} = V(r)\frac{\text{ModD}}{10000}, \tag{1}$$

where V is the market value of liability and r is the interest rate. ModD is the modified duration and is defined as:

$$\text{ModD} = -\frac{1}{V(r)}\frac{\partial V(r)}{\partial r}. \tag{2}$$

The second order Taylor expansion of the market value of the liabilities with respect to the change in yield is given by the following expression,

$$V(r+\varepsilon) = V(r) + \frac{\partial V}{\partial r}\varepsilon + \frac{1}{2}\frac{\partial^2 V}{\partial r^2}\varepsilon^2 + O(\varepsilon^3). \tag{3}$$

The focus of this thesis is on the first order hedging that is only with respect to the change of the market value of the liabilities based on the change in the interest rate. The second order sensitivity, the convexity, is omitted in further calculations. By reorganizing the terms in Equation 3, and ignoring the second order terms, the following relationship is obtained:

$$V(r)\Big(-\frac{1}{V(r)}\frac{\partial V(r)}{\partial r}\Big)\varepsilon = V(r) - V(r+\varepsilon) + O(\varepsilon^2). \tag{4}$$

Furthermore, by combining the left hand side of Equation 1, 2 and 4 for $\varepsilon = \frac{1}{10000}$ the following relationship is obtained:

$$V(r) - V\left(r + \frac{1}{10000}\right) =$$
$$= V(r)\cdot\text{ModD}\frac{1}{10000} + O\left(\left(\frac{1}{10000}\right)^2\right) =$$
$$= \text{DV01}(r) + O(10^{-8}).$$

Finally, following approximation holds:

$$\text{DV01}(r) \approx V(r) - V\left(r + \frac{1}{10000}\right). \tag{5}$$

Furthermore, the approximation may be extended to the multi-dimensional real-space, with $\bar{r}$ denoting the swap interest rate vector, that is $\bar{r} \in \mathbb{R}^N$.

$$\text{DV01}(i) \approx V(\bar{r}) - V(\hat{r}_i) \tag{6}$$

where $\hat{r}_i = \bar{r} + \frac{1}{10000}\cdot I_i$, with

$$I_j = \begin{cases} 1, & \text{if } j = i \\ 0, & \text{if } j \neq i \end{cases} \tag{7}$$

and $i \in \{1, 2, \ldots, LLP\}$. In other words, the $\bar{r}$ is the actual market swap rate curve at a given point in time and $\hat{r}_i$ is the shocked curve with $+1$ bp at a given term i. The approximation of DV01[10] will be used throughout the rest of the thesis.

The DV01s are plotted in Figure 8 and 9 for SEK denominated interest rate swap per 27 December 2013. The particularity, with the Smith-Wilson extrapolation methodology, for short *LLP*s can be observed by studying the DV01 bars. An increased interest rate should by the concept of "time value of money" give a decrease of the market value V, due to the "harder" discounting, where the discount factor for a cash flow at tenor T, defined as $d(T) = \frac{1}{(1+r)^T} > \frac{1}{(1+r+\varepsilon)^T}$ if $\varepsilon > 0$. Hence the present value of the future cash flow should decrease. In other words, if one is obliged to pay out a predetermined amount in the future less money needs to be invested today, in an interest-bearing instrument, if the interest rate have increased, in order to meet the future obligation. The present value of this future obligation should therefore decrease. One would therefore expect the *DV01* to be positive for all tenors i in the figures below. But DV01(7) and DV01(9) are negative for Company A, also DV01(5), DV01(7) and DV01(9) for Company B. This effect of alternating signs will be defined as the *peculiar Smith-Wilson effect* throughout this thesis.

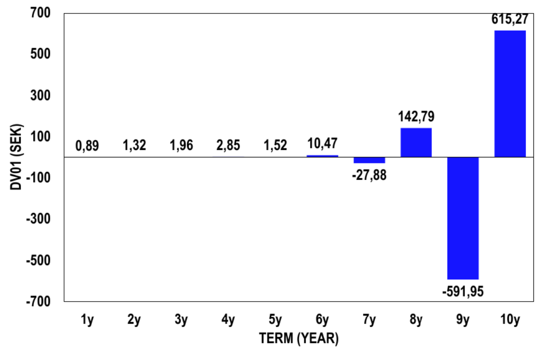

Figure 8. Dollar duration for Company A

In Figure 10 Company A's DV01s have been plotted for different $LLP \in \{10, 12, 15, 20, 30\}$ years. Notice that the *peculiar Smith-Wilson effect* vanishes completely at $LLP = 30$ year, and all DV01s have the same positive sign. The bars also decrease in magnitude when the *LLP* is increased, which is natural, since there are more points to tune the curve before the synthetic extrapolation phase starts beyond the *LLP*.

1.2.3 Setting up the hedge portfolio

The aim is to acquire a portfolio of bonds that is able to hedge the first order interest rate movement of the company's liabilities. The hedge portfolio is composed of bonds with

[10] Page 90 in [2] is recommended for further reading on the modified duration and the dollar duration

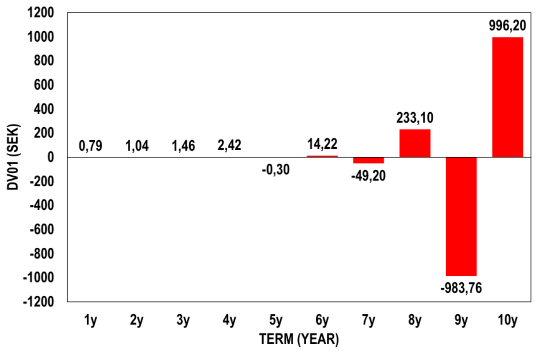

Figure 9. Dollar duration for Company B

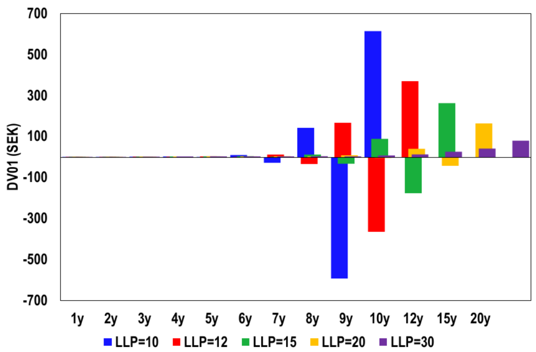

Figure 10. DV01 for Company A for different *LLP*s and $CP = 60$ years.

coupon frequency of 1 year. Bonds are also assumed to have whole year tenors up to the *LLP*.

The common way to hedge an interest rate liability is by selling derivatives known as interest rate swaps. Unlike bonds a swap agreement has zero entering cost. By selling a swap the company is assumed to acquire a predefined fixed interest rate from its counterpart with an annual coupon frequency. Hereafter, company obliges itself to pay out a floating interest rate to its counterpart. The floating coupon has a quarterly frequency and is typically based on the 3 month STIBOR in the Swedish market. Conversely, by buying a swap contract one would in turn receive the floating interest rate in exchange for fixed. The fixed leg of the swap is traded at par. A company's hedge portfolio will, in this thesis, be constructed out of bonds, in order to quantify how expensive the company's liabilities are to hedge. But since the coupon frequency is chosen to mimic the fixed leg of a swap agreement the results are transmittable to the common case when the hedge is constructed out of swaps.

Strategy The hedge strategy is to DV01 match the liabilities with assets, by acquiring bonds with whole year tenors up to the *LLP*, and reweigh the hedge portfolio every month. This is done by selling the hedge portfolio acquired during the previous month at its current market price, and acquiring a set of new bonds in order to DV01 match the liability at its current valuation.

Steady-state The liability portfolio is assumed to be at *steady-state*, that is intake of the business resets the cash flow profile of the liabilities at the beginning of every projection period to the original profile.

Self-financed The hedge portfolio is self-financed if the company needs to hold less assets in the hedge portfolio, that is the sum of the hedge portfolio's nominal amounts, than the market value of the company's liabilities.

Monthly historical SEK denominated swap interest rates from 25 January 2008 until 31 of January 2014 was used in the analysis, with the following terms in the data set: $\{1-10, 12, 15, 20, 30\}$.

A couple of simplifications have been made, since the focus of the thesis is mainly the sensitivity of a hedge portfolio based on interest movement and parameter choice of the valuation methodology: The hedge portfolio and the company's liabilities are both valued with the same yield, derived from Smith-Wilson methodology, with the average of bid and ask swap rates as input. In other words, the bid–ask spread have not been taken into account in the evaluation of the hedge's performance.

The yield environment that prevailed during the period is depicted in Figure 11.

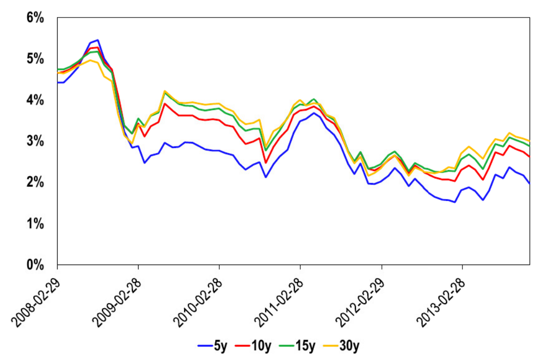

Figure 11. SEK denominated swap interest rates for a subset of tenors used in the analysis

Strategy that ensures that a change in interest rates will not affect the value of a portfolio are in literature referred to as *interest rate immunization* or *immunization of cash flows*, see example [6]. In this thesis cash flows are valued with Smith-Wilson methodology, hence the immunization is targeting the change in the underlying swap rates, and not directly the change in zero-coupon rate. The hedge portfolio is also constructed of a set of synthetic bonds that are passable on the market and constrained up to the *LLP* tenor.

2. DV01 matching and effects of different CP and LLP parameters

In this section, the performance of the hedge has been analyzed for a different choice of *LLP* and *CP* parameters.

2.1 Performance of the hedge strategy

The performance of the hedge portfolio has been evaluated by introducing three measures **Hedge–Liability ratio**, **Net Profit and Loss** and **Net cash flow**. The first measure explains how self-financed the hedge strategy is, while the following two measure explains the hedge portfolio's financial performance: **Net Profit and Loss** corresponds to the results of the company's Profit and Loss account, and the **Net cash flow** corresponds to the movements between the asset and liability side in the company's balance sheet.

- **Hedge–Liability ratio**: The assets which company needs to hold in the hedge portfolio in order to be DV01 matched, that is the sum of the nominal amounts for the different tenors, divided by the market value of the liabilities. In turn, if the **Hedge–Liability ratio** is below 100 percent the hedge strategy is *self-financed*.
- **Net Profit and Loss**: The difference between the **Hedge's P&L** and **Liability's P&L**, where
  - **Hedge's P&L** is today's market value of the bonds that were acquired, for the hedge portfolio, last month (new yield and reduced time to maturity) minus the market value of those bonds last month.
  - **Liability's P&L**: Company's cash flows from the previous month, which are now one month closer to maturity, discounted with today's yield minus the market value of the liabilities one month ago (that is original cash flow pattern discounted with the last month's yield).
- **Net cash flow**: **Liability cash flow** minus the **Hedge cash flow**, where
  - **Liability cash flow** is the market value of the new liability cash flows, coming from the new business, minus the pay-out of claims during the period
  - **Hedge cash flow** is the market value of today's hedge portfolio, acquired to match the liabilities at the current valuation, minus today's market value of the hedge portfolio that was acquired at the previous period

A mathematical representation of the hedge portfolio as well as its performance measures is provided in Section 5.1.

2.2 DV01 matching

2.2.1 Default parameters

The performance of the hedge strategy with EIOPA's default parameter setting, $LLP = 10$ years and $CP = 60$ years, for company A and B is illustrated in this section. Company A's market value of liabilities and the hedge portfolio is plotted in the Figure 12. Their relative relationship, i.e. the *Hedge–Liability ratio* is plotted in Figure 13. It is fairly cheap to

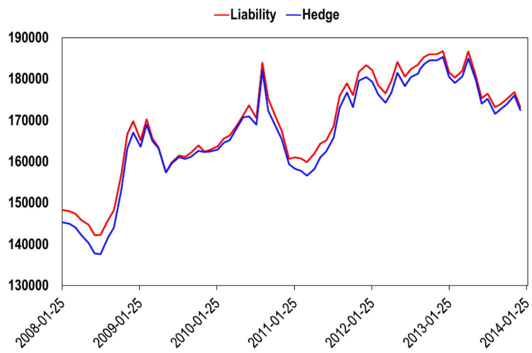

Figure 12. Company A: Liabilities and hedge portfolio for $(LLP, CP) = (10, 60)$ years

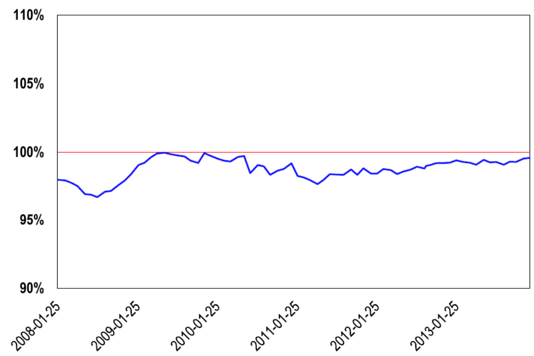

Figure 13. Company A: *Hedge–Liability ratio* for $(LLP, CP) = (10, 60)$ years

hedge the Company A's liabilities according to this strategy since the relative difference is strictly below 100% in this projection. This means that Company A needs to invest less assets into this hedging strategy than it holds in liabilities, for every time step of the projection, making the hedge *self-financed*. The performance of the Company A's hedge is illustrated in Figure 14. Company B's market value of liabilities and hedge portfolio is plotted in Figure 15, the *Hedge–Liability ratio* is plotted in Figure 16 and the performance of the hedge may be found in Figure 17.

It is more expensive to hedge Company B's profile, com-

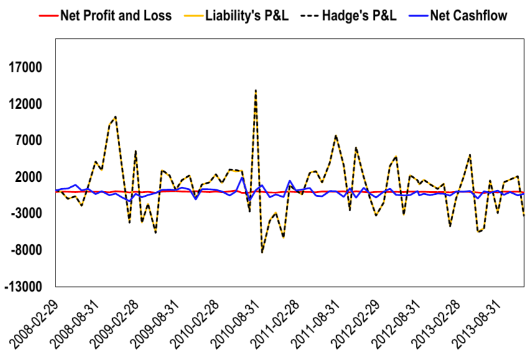

Figure 14. Company A: Hedge performance for $(LLP, CP) = (10, 60)$ years

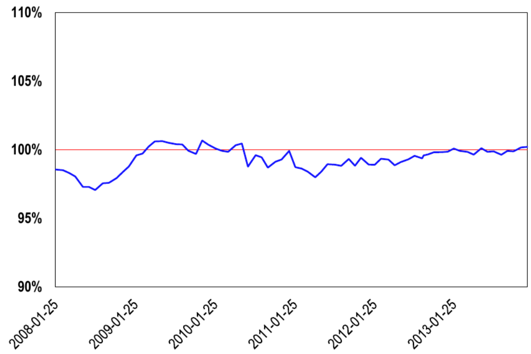

Figure 16. Company B: *Hedge–Liability ratio* for $(LLP, CP) = (10, 60)$ years

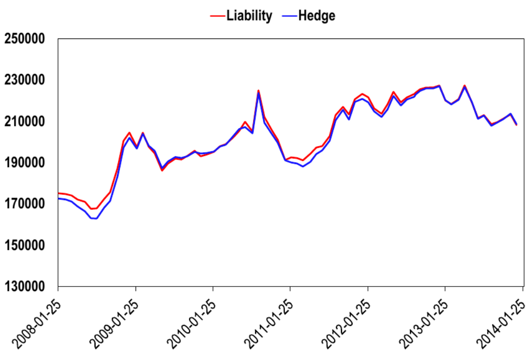

Figure 15. Company B: Liabilities and hedge portfolio for $(LLP, CP) = (10, 60)$ years

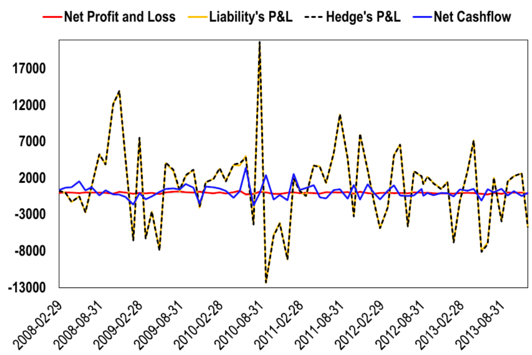

Figure 17. Company B: Hedge performance for $(LLP, CP) = (10, 60)$ years

pared to Company A's profile, since the market value of Company B's liabilities have a higher duration and are more sensitive to the movement of the swap price, in the calibration phase of the Smith-Wilson method.

2.2.2 Different *LLP* parameters

In Figure 18 and 19 the market value of the liabilities are illustrated for different *LLP* parameters. Moreover, in Figures 20 and 21 the *Hedge–Liability ratio* is illustrated. Conclusively, the hedge strategy is almost self-financed besides couple of spikes that appears for $LLP = \{12, 15, 20\}$, while being smooth for $LLP = \{10, 30\}$, for this projection period.

The *Net Profit and Loss* and *Net cash flow* during the projection period have been plotted in the histograms. Figures 22–26 for the Company A and Figures 27–31 for the Company B. The number of bins has been set to 200 for both *Net Profit and Loss* and *Net cash flow*. The *Net Profit and Loss* and *Net cash flow* are distributed around zero and implying an average a zero result. In contrast to the *Hedge-Liability ratio*, there are no patterns suggesting that no one choice of *LLP* parameter dominates another.

Table 3 and 4 illustrates how Company A and B should on average allocate their hedge capital, into bonds at different tenors, throughout the projection. The plus sign in the table indicates that the company needs to take a long position, in other words, receive the fixed interest rate. A company can do that by either purchasing a bond or entering a swap agreement by receiving the fixed interest rate and paying with floating interest rate. The minus sign on the other hand indicates that the company needs to short a bond on the market, or enter a swap agreement by receiving floating interest rate and paying the fixed interest rate. Moreover, in line with the liability DV01 pattern that can be observed in Figure 10 the hedge amount in the table also has an alternating sign pattern for *LLP*'s shorter than 30 years. Notice also the large leverage ratio: for example Company A needs on average to go short with 448.6% of their hedge amount into 9 year tenor while going long with 425.5% into the 10 years tenor in order to hedge their liability for the $LLP = 10$ years case, when

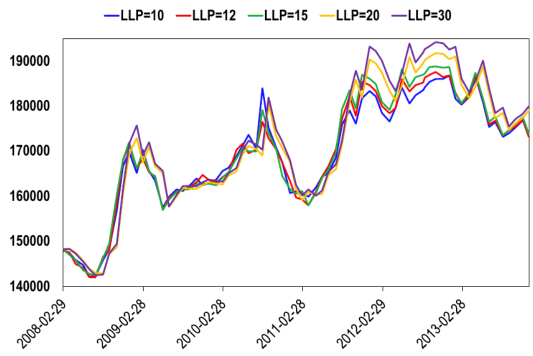

Figure 18. Company A: Market value of the liabilities for $CP = 60$ years

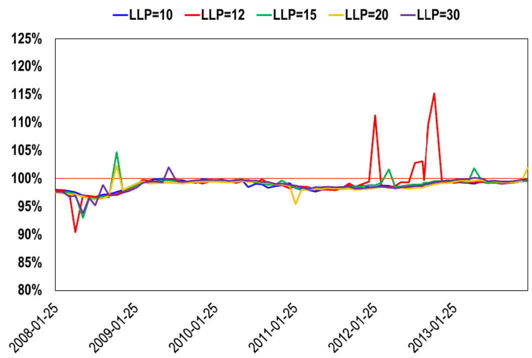

Figure 20. Company A: *Hedge–Liability ratio* for $CP = 60$ years

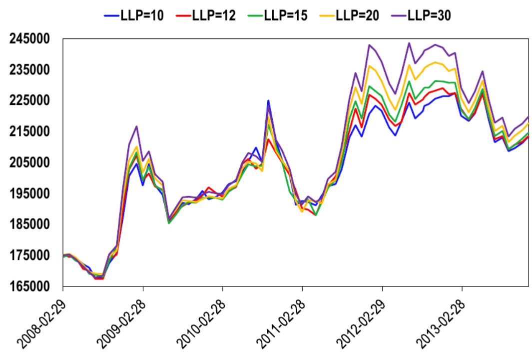

Figure 19. Company B: Market value of the liabilities for $CP = 60$ years

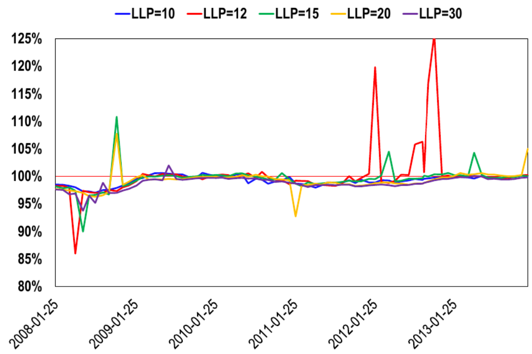

Figure 21. Company B: *Hedge–Liability ratio* for $CP = 60$ years

hedging with bonds.

Furthermore, the average of standard deviations decreases with an increased *LLP* parameter with values dropping from 7.8% with $LLP = 30$ years to 0.3% for $LLP = 10$ years for Company A. The similar drop is also observed for Company B. Implying that the hedge portfolio becomes more stable over time for a long *LLP* parameter. Which in practice leads to cost reduction for a hedge portfolio since it needs to be reweighed less frequently.

Conclusively, is has been observed that the hedge strategy is more or less self-financed for different choice of *LLP* parameter. Neither is the discussed performance measures sensitive to a change in the *LLP* parameter, returning zero result on average. The alarm goes off when one considers the practical aspects for an insurer to implement this simple hedge strategy. The Swedish long-term bond and swap market is small compared to the amount of SEK denominated insurance and pension liabilities, market value of which are estimated to 1 900 000 million SEK in [7]. The current valuation methodology will inevitably affect the market if the Swedish life-insurers would try to reduce their interest rate exposure.

2.2.3 Different *CP* parameters

The Smith-Wilson method's sensitivity towards the choice of the *CP* parameter will be discussed in this section. The *CP* parameter was finally set to 60 years for SEK denominated liabilities. But EIOPA have previously mentioned *CP* of 20 or 50 years as possible candidates. Hence the analysis in this section has been restricted to those three candidates, of *CP* parameter, mentioned above.

In Figure 32 the market value of Company A's liabilities are plotted. Notice that the analysis, in this section, will be restricted to Company A's liabilities due to the obvious trend that was observed when the *CP* parameter was reduced. Moreover, notice the small difference in market value of liabilities when the *CP* is set to the 50 or 60 years. In the beginning of the projection period, when the swap rates ware high, low market values were observed due to the "harder" discounting. In the second half of the projection period, when the swap

Table 3. Company A: Average weight per tenor of the total hedge value for $CP = 60$ years

(%)	Average					Standard deviation				
Ternor*LLP*	10y	12y	15y	20y	30y	10y	12y	15y	20y	30y
1 yr	5.1	5.1	5.1	5.1	5.0	0.2	0.2	0.2	0.1	0.1
2 yr	3.8	3.7	3.7	3.7	3.7	0.3	0.2	0.2	0.2	0.2
3 yr	3.8	4.0	3.9	3.9	3.9	0.3	0.2	0.2	0.2	0.1
4 yr	4.3	3.6	3.8	3.7	3.7	0.2	0.2	0.2	0.2	0.1
5 yr	1.8	4.2	3.6	3.7	3.7	0.4	0.1	0.2	0.1	0.1
6 yr	11.2	1.4	4.0	3.4	3.5	0.4	0.4	0.1	0.2	0.1
7 yr	−26.4	11.4	1.6	3.6	3.3	2.5	0.8	0.4	0.1	0.1
8 yr	119.5	−27.7	10.4	2.6	3.6	8.8	3.7	0.8	0.3	0.1
9 yr	−448.6	124.7	−23.6	6.7	2.7	35.1	13.7	3.7	0.7	0.1
10 yr	425.5	−248.0	60.1	−3.1	5.2	29.8	29.5	7.2	1.8	0.1
12 yr		217.6	−101.3	24.7	7.9		19.8	15.0	3.2	0.3
15 yr			128.8	−23.2	12.8			12.0	7.3	0.5
20 yr				65.1	16.3				6.5	0.8
30 yr					24.7					1.6
$\sum$	100.0	100.0	100.0	100.0	100.0					
Average						7.8	6.3	3.3	1.6	0.3

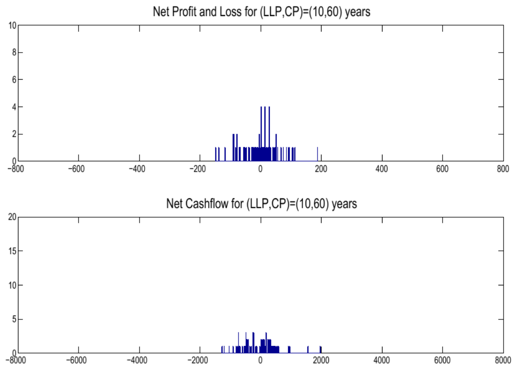

Figure 22. Company A: Histogram over the hedge performance for $(LLP, CP) = (10, 60)$ years

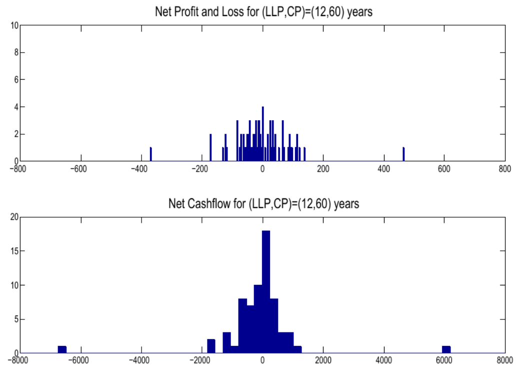

Figure 23. Company A: Histogram over the hedge performance for $(LLP, CP) = (12, 60)$ years

rates had decreased, the market values of liabilities are larger. The variance, of the market value of liabilities, is decreasing with a decreased *CP* due to a faster convergence towards the *UFR*.

The lower variance of the market value of the liabilities may at first be interpreted as an advantage for an insurance company, but from the Figure 33 large jumps in the *Hedge–Liability ratio* may be observed with decreased *CP*.

Conclusively, if the *CP* is reduced the market value of liabilities appears more stable on the company's balance sheet but becomes hard to hedge in practice due to the unstable hedge amount which in turn indicates for an increase in the leverage ratio. In the Table 5 one will see Company A's average *Leverage ratio* and its standard deviation (StdDev) throughout the projection. The *Leverage ratio* is also known as the debt–to–equity ratio and in this context measures the market value of liability plus the absolute value of the hedge positions for which the company is selling fixed interest rate (and therefore issues debt) divided by hedge positions where the company is buying fixed interest rate. One can observe that the standard deviation of the *Leverage ratio* increases with decreased *CP*.

Table 5. Company A: Leverage ratio and standard deviation for $LLP = 10$ years

CP	Average	StdDev
20 yr	97.87%	12.59%
50 yr	100.00%	3.25%
60 yr	100.21%	0.13%

Table 4. Company B: Average weight per tenor of the total hedge value for $CP = 60$ years

(%)	Average					Standard deviation				
Tenor\\LLP	10y	12y	15y	20y	30y	10y	12y	15y	20y	30y
1 yr	3.7	3.7	3.7	3.7	3.7	0.2	0.2	0.2	0.1	0.1
2 yr	2.4	2.4	2.4	2.4	2.3	0.3	0.3	0.3	0.2	0.2
3 yr	2.3	2.5	2.5	2.5	2.4	0.3	0.2	0.2	0.2	0.2
4 yr	3.0	2.1	2.3	2.3	2.2	0.3	0.3	0.2	0.2	0.2
5 yr	−0.5	3.0	2.0	2.2	2.2	0.6	0.2	0.3	0.2	0.2
6 yr	12.6	−0.9	2.8	2.0	2.1	0.6	0.7	0.2	0.3	0.2
7 yr	−38.5	14.0	−0.5	2.8	2.3	3.9	1.4	0.6	0.1	0.2
8 yr	161.6	−43.0	13.5	0.6	2.6	13.8	6.5	1.5	0.6	0.1
9 yr	−617.2	179.6	−40.2	9.6	1.8	54.8	23.8	6.7	1.6	0.2
10 yr	570.4	−364.9	91.6	−12.1	4.0	46.0	51.3	13.2	3.8	0.2
12 yr		301.5	−170.5	36.2	3.8		34.0	27.8	6.5	1.0
15 yr			190.5	−58.3	11.4			21.6	15.6	1.1
20 yr				106.1	12.1				13.2	2.9
30 yr					47.2					4.1
Σ	100.0	100.0	100.0	100.0	100.0					
Average						12.1	10.8	6.1	3.3	0.8

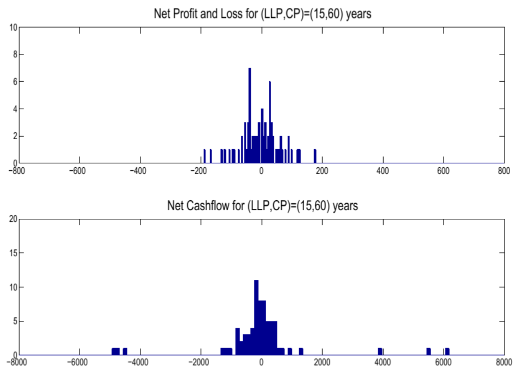

Figure 24. Company A: Histogram over the hedge performance for $(LLP, CP) = (15, 60)$ years

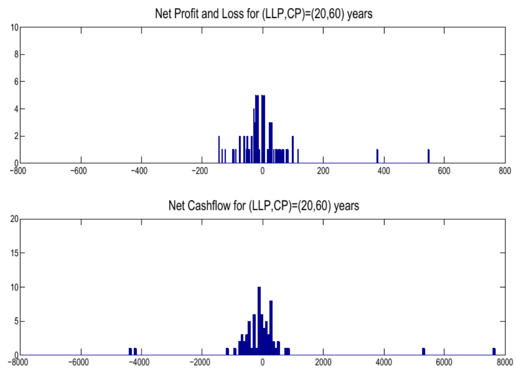

Figure 25. Company A: Histogram over the hedge performance for $(LLP, CP) = (20, 60)$ years

3. Constrained hedging

An alternative methodology for choosing the nominal hedge amounts is proposed in this chapter, allowing to constrain the positions to receive fixed interest rate only. Consider the *Net Profit and Loss* measure expressed as:

$$\begin{aligned} R(t) &= \Delta R_H(t) - \Delta R_L(t) = \\ &= (R_H(t) - R_H(t-1)) - (R_L(t) - R_L(t-1)). \end{aligned}$$

Moreover, the following ansatz is proposed:

$$\Delta R_H(t) = f(\Delta r_1(t), \Delta r_2(t), \ldots, \Delta r_{LLP}(t)) \tag{8}$$

and

$$\Delta R_L(t) = g(\Delta r_1(t), \Delta r_2(t), \ldots, \Delta r_{LLP}(t)) \tag{9}$$

for some functions $f, g \in R^1$ and $\Delta r_i(t) = r_i(t) - r_i(t-1)$, where $i \in \{1, 2, \ldots, LLP\}$ with t denoting the time step. Moreover, by applying the following approximative ansatz for f and g functions:

$$f(\Delta r_1(t), \ldots, \Delta r_{LLP}(t)) \approx a_0 + \sum_{i=1}^{LLP} a_i \Delta r_i(t) \tag{10}$$

$$g(\Delta r_1(t), \ldots, \Delta r_{LLP}(t)) \approx b_0 + \sum_{i=1}^{LLP} b_i \Delta r_i(t). \tag{11}$$

In turn, the following relationship is obtained:

$$R(t) \approx a_0 - b_0 + \sum_{i=1}^{LLP} (a_i - b_i) \Delta r_i(t). \tag{12}$$

Furthermore, the hedging algorithm is to minimize the variance of the *Net Profit and Loss*:

$$Var(R(t)) \approx \sum_{i=1}^{LLP} \sum_{j=1}^{LLP} (a_i - b_i)(a_j - b_j) Cov(\Delta r_i, \Delta r_j) \tag{13}$$

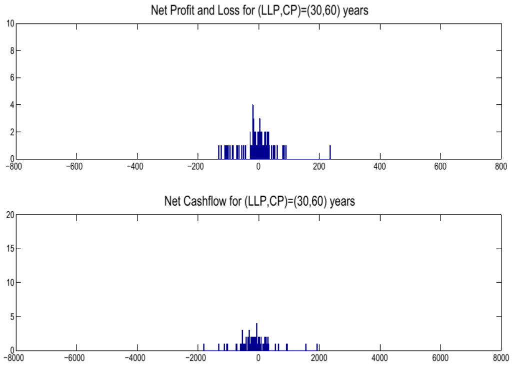

Figure 26. Company A: Histogram over the hedge performance for $(LLP,CP) = (30,60)$ years

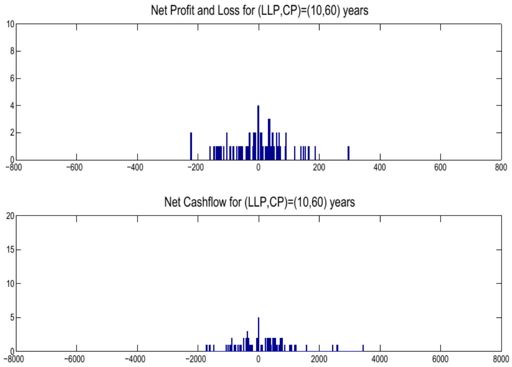

Figure 27. Company B: Histogram over the hedge performance for $(LLP,CP) = (10,60)$ years

In the matrix notation the relationship above may be expressed on the quadratic form:

$$Var(R(t)) \approx (\bar{a}-\bar{b})^T C(\bar{a}-\bar{b}) = \\ = \bar{a}^T C\bar{a} - 2\bar{a}C\bar{b} + \bar{b}C\bar{b},$$

where

$$\bar{a}(t) = (a_1(t), a_2(t), \ldots, a_{LLP}(t))^T$$
$$\bar{b}(t) = (b_1(t), b_2(t), \ldots, b_{LLP}(t))^T$$
$$C(t) = Cov(\Delta\hat{r}_i, \Delta\hat{r}_j).$$

The $\Delta\hat{r}_i$ is an estimator of the difference between two subsequent historical periods of the yield for a given tenor i:

$$\Delta r_i = \Big(r_i(t-N(t)) - r_i(t-N(t)+1), \\ r_i(t-N(t)+1) - r_i(t-N(t)+2), \ldots, \\ r_i(t) - r_i(t-1)\Big)^T$$

where $i \in \{1, 2, \ldots, LLP\}$. In this thesis the integer $N(t) = 36+t$, for the monthly projection period $t \geq 1$. This definition creates a window of historical data of 3 years from the

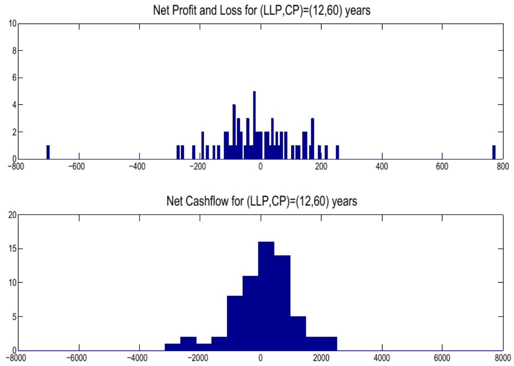

Figure 28. Company B: Histogram over the hedge performance for $(LLP,CP) = (12,60)$ years

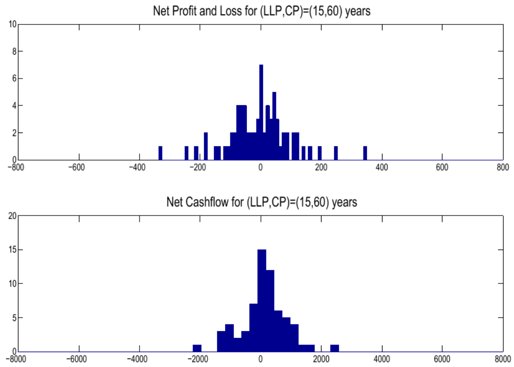

Figure 29. Company B: Histogram over the hedge performance for $(LLP,CP) = (15,60)$ years

beginning of the projection. Throughout the projection the window is extended one month at the time. Notice that the algorithm does not utilize any data ahead of time.

Since $\bar{b}^T C\bar{b}$ is a constant, with respect to $\bar{a}$, the optimization problem is hence to find $\bar{a} = (a_1, a_2, \ldots, a_{LLP})^T$ that minimizes equation:

$$\min_{\bar{a}} \left(\bar{a}^T C\bar{a} - 2\bar{a}C\bar{b}\right), \tag{14}$$

and it will be denoted as *minR*, throughout the rest of the thesis.

Furthermore, it is interesting to see how well this algorithm performs when the optimization problem described by

Table 6. Company A: The *Net Profit and Loss* divided by the *Market Value of Liabilities* for $(LLP,CP) = (10,60)$ years

Strategy	Min	Average	Max	StdDev
DV01 matched	-8.51 bp	-0.77 bp	5.17 bp	2.84 bp
minR	-8.51 bp	-0.77 bp	5.17 bp	2.84 bp
minR, a > 0	-108.91 bp	-0.67 bp	64.19 bp	36.81 bp

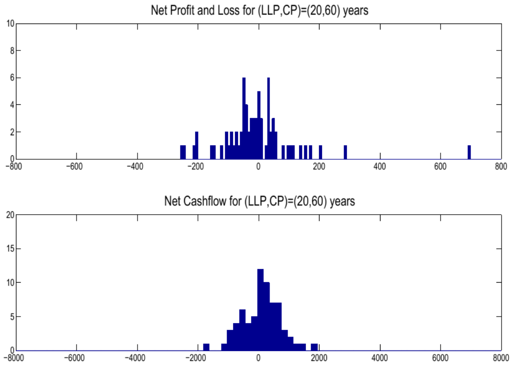

Figure 30. Company B: Histogram over the hedge performance for $(LLP,CP) = (20,60)$ years

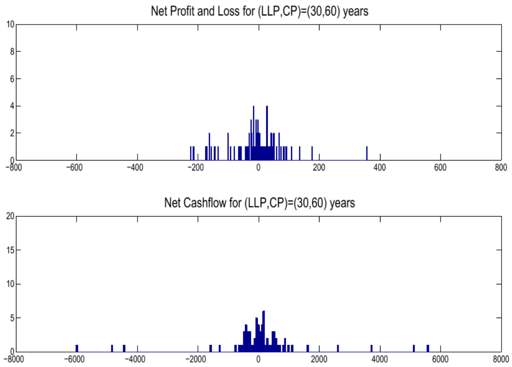

Figure 31. Company B: Histogram over the hedge performance for $(LLP,CP) = (30,60)$ years

14 is solved as subject to the following constraint:

$$(a_1, a_2, \ldots, a_{LLP})^T \geq (0,0,\ldots,0)^T, \tag{15}$$

that is hedging by allocating assets in long fixed interest rate positions only. The constrained optimization problem will be denoted as *minR*, $a > 0$, throughout the rest of the thesis.

Analysis in this chapter is restricted to Company A and default parameters, $LLP = 10$ years and $CP = 60$ years. In figures 34 and 35 the performance of the algorithms are plotted in terms of *Net Profit and Loss* and *Net Cash flow*.

The *minR* and *DV01 matched* strategies selects identical hedge amounts, not surprisingly, since the goal is the same in both set-ups. Moreover, one could conclude that there are no significant difference in performance of the *Net Cash flow* measure for this three set-ups. The *minR*, $a > 0$ set-up is clearly suboptimal with respect to *Net Profit and Loss* measure due to the constraint. Moreover, it can be observed from Tables 6 and 7 that the volatility in performance measures are modest in relationship to the market value of the liabilities. The *Hedge–Liability ratio* and the *Leverage ratio* are depicted in the Figure 36 and 37. Under the *minR*, $a > 0$ set-up, with restriction to long positions in fixed interest rates, the company

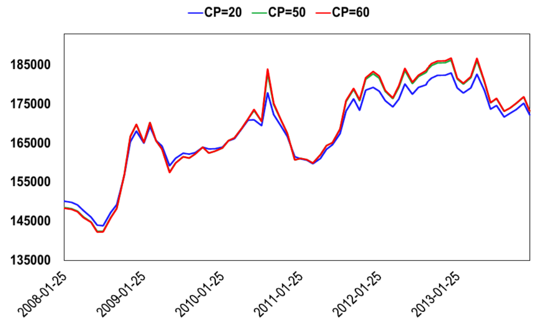

Figure 32. Company A: Market values of the liabilities for $LLP = 10$ years

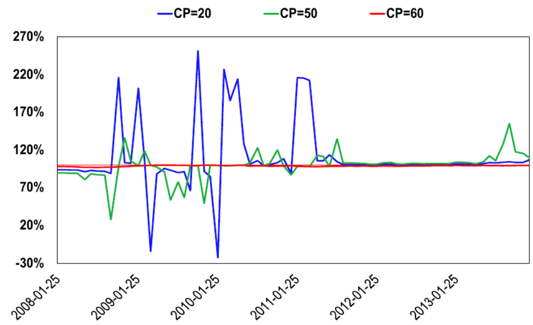

Figure 33. Company A: *Hedge–Liability ratio* for $LLP = 10$ years

is forced to hold a higher hedge amount and becomes more leveraged.

4. Key summary

This thesis presents an analysis of how the various choices of the *LLP* and the *CP* parameters affect the performance of a hedge portfolio, which aim to DV01 match long-tailed life-insurance and pension liabilities valued by EIOPA's Smith-Wilson methodology.

Firstly, it has been demonstrated that a short choice of the *CP* parameter leads to decreased variance of the liabilities market value, but at the same time to a deteriorated performance of the hedge portfolio.

Secondly, the analysis revealed that the choice of the *LLP*

Table 7. Company A: The *Net Cash flow* divided by the *Market Value of Liabilities* for $(LLP,CP) = (10,60)$ years

Strategy	Min	Average	Max	StdDev
DV01 matched	-50.70 bp	-6.48 bp	96.49 bp	28.03 bp
minR	-50.70 bp	-6.48 bp	96.49 bp	28.03 bp
minR, $a > 0$	-106.12 bp	-2.83 bp	92.58 bp	47.81 bp

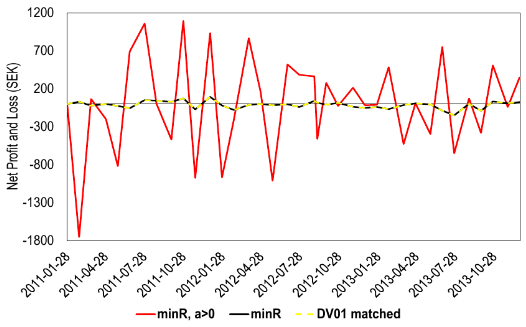

Figure 34. Company A: Net Profit and Loss for different hedge algorithms $(LLPCP) = (10, 60)$ years

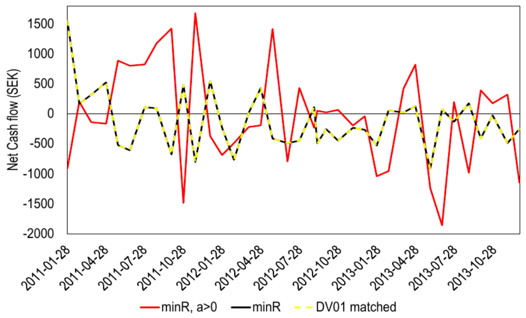

Figure 35. Company A: Net Cash flow for different hedge algorithms $(LLP, CP) = (10, 60)$ years

parameter has a small effect on the financial performance of a theoretical hedge portfolio. On the other hand, *LLP*'s 20 years and below will force companies to reduce their interest rate risk exposure by selling fixed interest rate at the tenor $LLP - 1$ and buying fixed interest rate at the tenor *LLP*, for a substantial amount of money, hence increasing companies leverage and challenging the market depth at these two points. Uncertainty in how much hedge capital a company needs to allocate to different tenors increases with reduced *LLP* parameter. This will in turn oblige companies to re-balance their hedge portfolios more frequently, driving the asset management cost. The question going forward for EIOPA is if they can have faith in the market to become more active at a higher *LLP* parameter or possibly leave the Swedish policy holder less protected against interest rate risk and with higher asset management costs.

Finally, this thesis proposes an alternative set-up of the hedge portfolio, allowing companies to constrain the asset allocation to long positions in fixed interest rates. It has been demonstrated that the performance of the constrained hedge portfolio is less self-financing and sub-optimal in terms of *Net Profit and Loss* performance measure. Still the alterna-

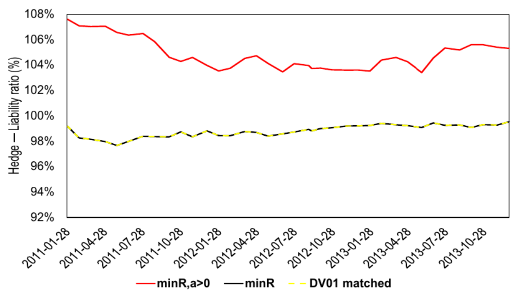

Figure 36. Company A: *Hedge–Liability ratio* for different hedge algorithms $(LLP, CP) = (10, 60)$ years

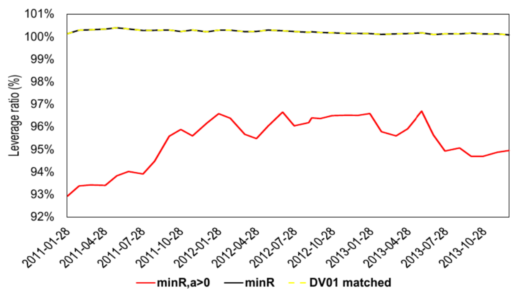

Figure 37. Company A: *Leverage ratio* for different hedge algorithms $(LLP, CP) = (10, 60)$ years

tive hedge portfolio set-up equips companies with a more practically framework for liability hedging under the current valuation methodology.

5. Appendix

5.1 Technicalities of the projection

5.1.1 DV01 matching

A bond's DV01 for the tenor i is denoted as $\text{DV01}_B(i)$ and calculated through the following Equation 16 based on results derived in Section 5.2. Notice that this DV01 is per 1 SEK.

$$\text{DV01}_{B,1\text{SEK}}(i) = \left(1 - \frac{c_i}{c_i + 1bp}\right)\left(1 - \frac{1}{(1 + c_i + 1bp)^i}\right) \quad (16)$$

for swap interest rate c_i at the tenor $i \in \{1, 2, \ldots, LLP\}$.

Moreover, the nominal amount that is invested in the hedge portfolio for a specific tenor bucket i is denoted $H(i)$:

$$H(i) = \frac{\text{DV01}_L(i)}{\text{DV01}_{B,1\text{SEK}}(i)} \quad (17)$$

for each tenor $i \in \{1, 2, \ldots, LLP\}$. This relationship simply states: how much assets should be invested in each bond with a certain tenor i, in order to hedge the liability's $\mathrm{DV01}_L$ of the tenor i, where i is allowed to take values up to the *LLP*.

5.1.2 Performance measures

The performance of the hedge portfolio is measured by the *Net Profit and Loss* is denoted R, and the *Net cash flow* is denoted C.

$$R(t) = R_H(t) - R_L(t) \tag{18}$$

In Equation 18 the R_H denotes the *Hedge's P&L* and R_L denotes the *Liability's P&L*.

$$\begin{aligned} R_H(t) &= \bar{H}_{t-1}^T \cdot \bar{P}(\bar{c}_{t-1}, \tilde{D}_t) - \bar{H}_{t-1}^T \cdot \bar{P}(\bar{c}_{t-1}, \bar{D}_{t-1}) = \\ &= \bar{H}_{t-1}^T \cdot \bar{P}(\bar{c}_{t-1}, \tilde{D}_t) - \sum_{i=1}^{LLP} \bar{H}_{t-1}(i), \end{aligned}$$

where $\bar{P}(\bar{c}_{t-1}, \tilde{D}_t)$ is the price vector of the bonds bought one month ago, thus the discount factor $\tilde{D}_t(i)$ is at today's yields but with tenors one month closer to maturity that is $i \in \{1 - \frac{1}{12}, 1 - \frac{1}{12}, \dots LLP - \frac{1}{12}\}$. $\bar{D}_{t-1}(i)$ denotes the discount vector at last month's yield and with discount factors' tenors $i \in \{1, 2, \dots, LLP\}$. Since the bonds are traded at par the following relationship holds $\bar{P}(\bar{c}_{t-1}, \bar{D}_{t-1}) = (1, 1, 1, \dots, 1)^T$, where T denotes the transpose of the vector. The vector $\bar{H}_{t-1}$ denotes the nominal amount invested in bonds at different tenors, and the vector $\bar{c}_{t-1}$ denotes the swap rate at different tenors; both vectors are at the previous period $t-1$.

More explicitly, the price of a bond in the hedge bucket i which was purchased one month ago is:

$$P_i(\bar{c}_{t-1}, \tilde{D}_t) = \tilde{D}_t(i) + \bar{c}_{t-1}(i) \cdot \sum_{j=1}^{i} \tilde{D}_t(n), \tag{19}$$

where $i \in \{1, 2, \dots, LLP\}$. The discount factors tenors' are reduced by one month to $\{1 - \frac{1}{12}, 1 - \frac{1}{12}, \dots LLP - \frac{1}{12}\}$, since the bonds are one month closer to maturity.

Moreover,

$$R_L(t) = \sum_{i=1}^{1200} F(i) \cdot d_t(i) - \sum_{i=2}^{1200} F(i) \cdot d_{t-1}(i). \tag{20}$$

$F(i)$ denotes the liability cash flow and $d_t(i)$ is the annual discount factor at a monthly tenor $i \in \{1, 2, \dots, 1200\}$, since the cash flow vector $\bar{F}$ is 1200 months long.

Furthermore,

$$C = C_L - C_H. \tag{21}$$

In Equation 21, C_L denotes the *Liability–cash flow* and C_H denotes the *Hedge–cash flow*.

$$C_L(t) = N(t) - F(1), \tag{22}$$

where N is the market value of liabilities coming from the new business. Since a steady-state is assumed it can be derived as

$$N(t) = \sum_{1}^{1200} F(i) \cdot d_t(i) - \sum_{2}^{1200} F(i) \cdot d_t(i). \tag{23}$$

Moreover,

$$\begin{aligned} C_H(t) &= \bar{H}_t^T \cdot \bar{P}(\bar{c}_t, \bar{D}_t) - \bar{H}_{t-1}^T \cdot \bar{P}(\bar{c}_{t-1}, \tilde{D}_t) = \\ &= \sum_{i=1}^{LLP} \bar{H}_t(i) - \bar{H}_{t-1}^T \cdot \bar{P}(\bar{c}_{t-1}, \tilde{D}_t). \end{aligned}$$

5.2 Dollar value of a basis point for a bond

In order to derive the Equation 16, take a bond with a principal amount of 1 SEK for which the yield to maturity is r for the coupon c which is paid out at an annual frequency. The time to maturity is given in a whole number of years: n. The price of this contract, V, has the following relationship:

$$\begin{aligned} V(c) &= \frac{1}{(1+r)^n} + c \sum_{k=1}^{n} \frac{1}{(1+r)^k} && (24) \\ &= \frac{1}{(1+r)^n} + \frac{c}{r}\left(1 - \frac{1}{(1+r)^n}\right). && (25) \end{aligned}$$

A bond is traded on par: $V(c) = 1$, if $r = c$. By using Equation 6 the following relationship is obtained

$$\begin{aligned} \mathrm{DV01} &\approx V(c) - V(c + \frac{1}{10000}) = \\ &= 1 - V(c + \frac{1}{10000}) = \\ &= 1 - \frac{c}{c + \frac{1}{10000}}\left(1 - \frac{1}{(1 + c + \frac{1}{10000})^n}\right) - \\ &\quad \frac{1}{(1 + c + \frac{1}{10000})^n} = \\ &= \left(1 - \frac{c}{c + \frac{1}{10000}}\right)\left(1 - \frac{1}{(1 + c + \frac{1}{10000})^n}\right). \end{aligned}$$

Acknowledgement

I would like to thank my supervisor Andreas Lagerås.

References

[1] Laura Rebel. The ultimate forward rate methodology to value pension liabilities: A review and an alternative methodology. *Cardano Rotterdam and VU University Amsterdam*, 2012.

[2] John C. Hull. *Options, futures and other derivatives*, volume Seventh edition. 2009.

[3] Finansinspektionen. Bedömning av aktiva marknader för fffs 2013:23. *Promemoria*, FI Dnr 13-11409:1, 2013.

[4] European insurance and occupational pension authority. Technical specefications part 2 on the long-term guarantee assesment. *EIOPA/12/307*, 2013.

[5] European insurance and occupational pension authority. Consultation papper on a technical document regarding the risk free interest rate term structure. *EIOPA-CP-14/042*.

[6] Henrik Hult; Filip Landskog; Ola Hammarlid; Carl Johan Rehn. *Risk and Portfolio Analysis*.

[7] Finansinspektionen. Förslag till nya regler om försäkringsföretags val avräntesats för att beräkna försäkringstekniska avsättningar. *Remisspromemoria*, FI Dnr 13.795:16, 2013.