STOCKHOLM UNIVERSITY
Department of Mathematics
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Examination in
Mathematics for Economic and Statistical Analysis
Master Program; 7,5 ECTS
30th September, 2015

Time: 09:00-14:00

Instructions:
- During the exam you CAN NOT use any textbook, class notes, or any other supporting material.
- Only **non-graphic** calculators are allowed during the exam.
- In all your solutions show your reasoning, explaining carefully what you are doing. JUSTIFY your answers.
- Use natural language, not just mathematical symbols.
- Use clear and legible writing. Write preferably with a ball-pen or a pen (black or dark blue ink).
- Mark clearly where is your final answer putting $\boxed{\text{A BOX}}$ around it.

Grades: Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

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1. Calculate the limits

$$\text{a)} \quad \lim_{x\to+\infty} \frac{1+x\ln x}{x+2\ln x} \qquad\qquad \text{b)} \lim_{x\to 1} \frac{6x^2+4x-10}{9x^2-1}$$

2. Consider the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$.
   (a) Find the critical points;
   (b) Find the intervals on which f is increasing and decreasing;
   (c) Give the maximum and minimum value of f on the interval $[0,3]$.

3. Calculate the integrals

$$\text{a)} \quad \int \frac{2x^3-2x^2+1}{x-1}\mathrm{d}x \qquad\qquad \text{b)} \quad \int_0^1 \frac{t\ln(t^2+1)}{t^2+1}\mathrm{d}t$$

4. The expression

$$x^2\ln y + y^3 e^{-x} = 8,$$

   defines y as a function of x: $y = y(x)$.
   (a) Find the value $y(0)$.
   (b) Find the equation of the tangent line to $y(x)$ at the point $P = (0, y(0))$.

5. Let $F(x,y) = x^4 + y^4 - 36xy$. Find all stationary points for this function and determine whether they are local maximums, minimums, or saddle points.

6. Compute the Taylor polynomial of degree 2 for the function $g(x) = \sqrt[3]{x+1000}$, centered at the point $a = 0$. Use this polynomial to find an approximate value of $\sqrt[3]{1003}$.

7. Solve the following system of linear equations using Cramer's Rule:

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & + & 4x_3 & = & 2 \\ & & -x_2 & - & 3x_3 & = & 1 \\ 2x_1 & + & 2x_2 & + & 6x_3 & = & 2 \end{array}$$

GOOD LUCK!

The corrected exams will be handed out on Wednesday, October 7 2015, at 10:30, in the room next to the coffee shop, house 5, and after that in room 204, house 6.