Six lectures about Riemann's ζ-function

The lectures will be devoted to the ζ-function which was introduced by Bernhard Riemann in his epoch-making article from 1859 given by the meromorphic function of a single complex variable s which in the half-space $\mathfrak{Re}(s) > 1$ is defined by the Dirichlet series

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

The ζ-function plays a central role in analytic number theory. In the lectures we shall for example prove the *Prime Number Theorem* which gives an asymptotic formula for the density of prime numbers in the set of positive integers. Basic acquaintance about analytic function of a single complex variable is a prerequisite. But I will make an attempt to present the material in a reasonably self-contained manner. A crucial result towards the Prime Number Theorem is for example the powerful *Tauberian Theorem* due to Ikehara which is presented in the lecture entitled *Beurling-Wiener algebras.*

Two lectures covering more advanced material expose results due to Arne Beurling. They provide support for the validity of the *Riemann Hypothesis* which in spite of extensive numerical verifications via computers remain as a challenging open problem in in mathematics.

Here follow titles of the individual lectures:

1. *The functions* $\Gamma(s)$ *and* $\zeta(s)$
2. *The Riemann hypothesis*
3. *Beurling-Wiener algebras*
4. *The prime number theorem.*
5. *A uniqueness result for the* ζ*-function.*
6. *Beurling's criterion for the Riemann hypothesis*

Notes covering the material above will be delivered during the course