Department of Mathematics
STOCKHOLM UNIVERSITY

Examiner: Paul Vaderlind

Examination in
Mathematics for Economic and Statistical Analysis
Master Program; 7,5 ECTS
October 2, 2013; time: 09:00–14:00

Only non graphic calculators are allowed.
Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

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1. There exists a **rational** number a such that $\dfrac{3x^3-5x^2-8x-2}{x-a}$ can be expressed as a quadratic polynomial Ax^2+Bx+C. Find a, A, B and C.

2. For which x is the series $\displaystyle\sum_{n=0}^{\infty}\Big(\frac{2x^2-3}{5}\Big)^n$ convergent? For which x is the sum equal $\dfrac{5}{6}$?

3. Determine if the given improper (generalized) integral exists, and evaluate it if it does:
$\displaystyle\int_1^2 \frac{2x}{\sqrt{x^2-1}}\,dx$.

4. (Implicit differentiation) The expression

$$x^2e^y+xy^3-(e^x+1)y+(x+1)^2-3=0$$

defines y as a function of x: $y=y(x)$.
(a) Find the value $y(0)$.
(b) Find the equation of the tangent line to $y(x)$ at the point $P=(0,y(0))$.

5. Use Cramer's rule to solve the following system of equations

$$\begin{cases} -2x+3y-z=-14 \\ 5x-2y+3z=19 \\ 4x+y-2z=3 \end{cases}$$

6. Let $h(x,y)=x^3y-\dfrac{1}{3}xy^3+9x-2013$. Find all stationary points for this function and determine whether they are local max-, min-, or saddle-points.

7. Consider $f(x)=\ln(x^2-1)$.
(a) For which x is $f(x)$ well-defined? Write your answer in form of intervals (a,b).
(b) Find the intersection points between the graph of $f(x)$ and the x-axis and y-axis.
(c) Determine the intervals where the function is increasing/decreasing.
(d) For each interval (a,b) where $f(x)$ is well defined (as in part (a)) find $\lim\limits_{x\to a^+} f(x)$ and $\lim\limits_{x\to b^-} f(x)$.

GOOD LUCK!

The papers will be handed out on Monday, October 7, 2013, at 12.30, in the room next to the Coffee Shop, house 5, and after that in room 204, house 6.