

**Time:** 09:00-14:00

**Instructions:**

- During the exam you CAN NOT use any textbook, class notes, or any other supporting material.
- Only **non-graphic** calculators are allowed during the exam.
- In all your solutions show your reasoning, explaining carefully what you are doing. JUSTIFY your answers.
- Use natural language, not just mathematical symbols.
- Use clear and legible writing. Write preferably with a ball-pen or a pen (black or dark blue ink).
- Mark clearly where is your final answer putting A BOX around it.

**Grades:** Each solved problem is awarded by up to 10 points. At least 30 points are necessary for the grade E, 36 for D, 42 for C, 48 for B and 54 for A. Note that the problems are not ordered according to the difficulty!

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1. Determine if the given improper integral exists, and if it does, evaluate it:  $\int_0^1 x^{2016} \ln x \, dx$ .

2. Let  $F(x, y) = yx^2 - y - 1$ .

- (a) Find all stationary points for this function and determine whether they are local maximum, minimum, or saddle points.
- (b) Define  $D = \{(x, y) : x \geq 0, y \geq 0, x + y \leq 1\}$ . Does it exist a maximum point of  $F$  on  $D$ ? Argument your answer. If so, find the maximum value of  $F$  on  $D$ .

3. The expression

$$\ln(1 + y^2x) + y = 1,$$

defines  $y$  as a function of  $x$ :  $y = y(x)$ .

- (a) Find the value  $y(0)$ .
- (b) Find the equation of the tangent line to  $y(x)$  at the point  $P = (0, y(0))$ .
- (c) Find the Taylor polynomial of order 2 of the function  $y(x)$  about  $x = 0$ .

4. Solve the following system of linear equations using Cramer's Rule:

$$\begin{array}{rcccc} 4x_1 & + & x_2 & + & 2x_3 & = & 2 \\ 9x_1 & + & 3x_2 & + & 3x_3 & = & 3 \\ 3x_1 & & & + & x_3 & = & -1 \end{array}$$

5. Which of the following statements are true and which are not? No reasoning needs to be given: just answer T (true) or F (false) for each of them. For each correct answer you will be awarded 1 mark. For each incorrect answer you will lose 1 mark. If the question is left unanswered, no penalty is applied. If the number of incorrect answers is higher than the number of correct answers, then the total mark awarded for this question will be 0.

i. The domain of definition of the function  $f(x) = \ln\left(\frac{1}{1-4x^2+4y^2}\right)$  is the set  $\{(x, y) : \sqrt{x^2 + y^2} \leq 1/2\}$ .

ii. The power series  $\sum_{k \geq 0} \left(\frac{1}{a^2-1}\right)^k$  converges for  $|a| \geq 2$ .

iii. The line  $y - 3 = 0$  is a horizontal asymptote of the function  $f(x) = 3x^2(x^2 + e^x)^{-1}$  as  $x$  tends to  $-\infty$ .

