SF2735/MM8020 Homological algebra and algebraic topology

Homework assignment 1

(1) (4pt) Let the polynomial ring $A = \mathbb{Q}[x]$ act $\mathbb{Q}$-linearly on $M = \mathbb{Q}^3$ by

$$x \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} q_1 + q_2 \\ q_1 + q_3 \\ q_2 + q_3 \end{bmatrix}$$

- (a) Show that this defines a unique A-module structure on M.
- (b) Find a minimal set of generators of M.
- (c) Show that M is not free.

(2) (3pt) Define a $\mathbb{Q}$-linear action of $R = \mathbb{Q}[t]$ on $S = \mathbb{Q}[x]$ by $t^n\, f(x) = \dfrac{\partial^n}{\partial x^n} f(x)$, for all $n \geq 0$.

- (a) Show that this defines an R-module structure on S.
- (b) Show that S is not finitely generated as an R-module.

(3) (3pt) Let $R = \mathbb{Q}[x, y, z]$ and let $\Phi\colon R^3 \longrightarrow R^2$ be given by the matrix

$$\begin{bmatrix} x & y & z \\ y & z & x \end{bmatrix}.$$

- (a) Determine a minimal generating set for $\ker \Phi$.
- (b) Determine whether $\ker \Phi$ is a free R-module.

Discussing the homework problem with each other is admissible and even encouraged, but you have to formulate your solutions separately. Such collaboration should be clearly declared in the homework of all the participants. Identical or nearly identical solutions or solutions copied from sources on the internet are not acceptable.

The solutions should be submitted by email to Matthias Grey (mgrey@math.su.se) as pdf no later than **Tuesday September 5 at 3pm**.