SF2735/MM8020 Homological algebra and algebraic topology

Homework assignment 2

(1) (4pt) Let $R = \mathbb{Z}/7$ and let $K_\bullet$ be defined by $K_i = R^3$ for all $i \in \mathbb{Z}$ and $d_i \colon K_i \longrightarrow K_{i-1}$ be given by the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$$

for all i.

(a) Verify that $K_\bullet$ is a complex of free R-modules.

(b) Compute the homology of $K_\bullet$.

(2) (3pt) Let $f \colon P_\bullet \longrightarrow Q_\bullet$ be a chain map of complexes. We define the *mapping cone* in the following way. Let $M_n = P_{n-1} \oplus Q_n$, and define $d_n^M \colon M_n \longrightarrow M_{n-1}$ by

$$d_n^M(x, y) = (-d_{n-1}^P(x), d_n^Q(y) + f_{n-1}(x)).$$

Show that (M_n, d_n^M) forms a complex, and that we have a long exact sequence

$$\cdots \longrightarrow H_n(Q) \longrightarrow H_n(M) \longrightarrow H_{n-1}(P) \longrightarrow \cdots .$$

(3) (3pt) Apply the mapping cone construction from the previous problem to the chain map $K_\bullet \longrightarrow K_\bullet$ where $K_\bullet$ is the complex from the first problem and the map $f_i \colon K_i \longrightarrow K_i$ is given by the matrix

$$\begin{bmatrix} -3 & -3 & -2 \\ 3 & -2 & 2 \\ 1 & -3 & 3 \end{bmatrix}$$

for all $i \in \mathbb{Z}$. In particular, give explicit descriptions of the maps of the long exact sequence

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Discussing the homework problem with each other is admissible and even encouraged, but you have to formulate your solutions separately. Such collaboration should be clearly declared in the homework of all the participants. Identical or nearly identical solutions or solutions copied from sources on the internet are not acceptable.

The solutions should be submitted by email to Matthias Grey (mgrey@math.su.se) as pdf no later than **Tuesday September 12 at 3pm**.