Stockholm University
Department of Mathematics
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4. Exercise "Data Structures and Algorithms (DA 4006)"

Problem 1: 4+2=6p

(a) Given is the following array $A = [2, 1, 1, 2, 0, 2]$. Apply `COUNTING-SORT`$(A, 6, 2)$ according to the lecture. Provide the array C after Line 4 and, after each step of the for-loop in Line 5, the arrays B and C.

(b) Provide an array A of length $n = 4$ for which the array C becomes $C = [0, 0, 0]$ after the run of `COUNTING-SORT`$(A, n, 2)$. Briefly explain your choice of A.

Problem 2: 4+ 4=8p

Given is the following sequence of numbers L stored in an array, sorted in ascending order:

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$L[i]$	19	31	44	52	91	108	111	119	127	130	150	175	196	201	202	203

i	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
$L[i]$	207	211	215	218	221	227	231	235	239	243	250	252	267	269	277	278

For the search algorithms

(a) Binary Search,

(b) Simple Jump Search with optimal jump width.

provide the search steps for searching for element 135. Apply the algorithms exactly as presented in the lecture. Describe all essential calculations and decisions.

Problem 3: 6p

Provide pseudocode for a non-recursive implementation of the algorithm `INORDER-TREE-WALK`(x) (see lecture slide 18 in part 3 searching) that uses the data structure *stack*. For the stack, use only the stack operations as defined in the lecture. Explain shortly how and why your algorithm works.

Problem 4: 6p

Given are the ordered sequences of keys

a) `1, 3, 5, 2, 4, 6`

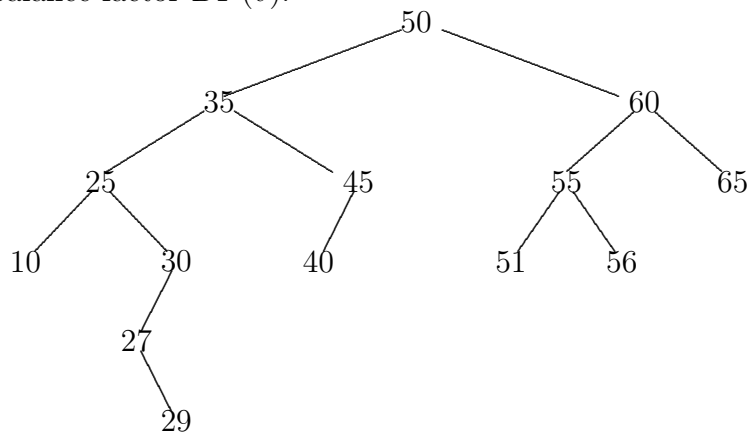
b) `5, 6, 4, 2, 3, 1`

c) `5, 3, 6, 4, 2, 1`

Build the binary search tree for each of the sequences. That is, insert the keys one after another into an initially empty binary tree. Provide a simple drawing of the tree after the last insertion. Then, delete the key `2` in each case and draw the resulting tree.

Problem 5: 4p

Given is the following binary search tree. Provide for each inner vertex v the respective balance factor $BF(v)$.



Problem 6: 4+4=8p

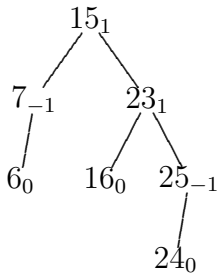
Construct an AVL tree for each of the key sequences (a) and (b) by inserting the keys in the given order into an initially empty AVL tree and performing any necessary rotations after each insertion. Carefully test after each insertion whether a rotation (or double rotation) is necessary.
For each rotation, draw the (unbalanced) trees *immediately before the rotation*, including the balance factors as indices of the inner vertices. Also, draw the overall *resulting tree*. Additionally, specify the *type of rotation*.
A double rotation is considered as two consecutive single rotations.

(a) 10, 9, 8, 7

(b) 10, 2, 11, 17, 6, 5, 4

Problem 7: 4p

Given is the following AVL tree T



including its balance factors as subscripts.

Delete the key 15 from T.

Describe the applied substeps of the AVL deletion operation (including any rotations) and illustrate the resulting AVL tree including balance factors at inner vertices after each single rotation.
A double rotation is considered as two consecutive single rotations.

Problem 8: 5p

Construct a Red-Black tree for the key sequence $30, 10, 5$ by inserting the keys in the given order into an initially empty Red-Black tree. For any insertion draw the tree before and after any recoloring or rotation.

Problem 9: 5p

Given is a hash table H of size $m = 7$. Access to H is done using Linear Probing.
The hash functions for keys k is here defined as follows:

$$h(k, i) = (k + 2 \cdot i) \mod m \qquad (i = 0, 1, 2, \ldots) \; .$$

Insert the keys `9,12,5,3,7` in this order into the initially empty hash table H; indicate the table and the respective value of i after each insertion step.

Problem 10: Bonus Exercise 3+3 = 6p

You, as the Hitchhiker, are using your "guide to the galaxy" and arrive at the kingdom of `asciopolis` located in the 16-base quadrant of the 128-base modules, where the boundaries between the digital and physical worlds blur and a profound mystery casts a shadow over the land. Deep within the heart of the city lies a sacred artifact, two hidden codes, said to hold the key to the kingdom's continued prosperity. However, legends foretell that failure to unlock its secrets will plunge the kingdom into a dark age, where the world regresses to a mere binary of black ones and white zeros.
The first code will give the answer to the ultimate question of life, the universe, and everything while the second code reveales the holy recipe of liquid bread.
You have your supercomputer Deep Thought with you. As you stand before the Council of the kingdom, your name is called forth and you are asked to reveal the two hidden codes.

Code 1 is:

```
54 68 65 20 61 6E 73 77 65 72 20 69 73 3A 20 34 32
```

Code 2 is:

```
240743138
```

As a computer scientist and with your supercomputer Deep Thought, you immediately know what to do and you spell out loud the correct meaning of the codes! Ahem, what are the unraveled codes again? Explain your solution.

Hint: the name and location of the kingdom may help.

Bonus problems can be used to earn a certain amount of extra points counted for the final exam.

Deadline: Thursday, May 14