

SUGGESTION FOR PROJECTS IN DERIVED CATEGORIES

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A list of various topics involving triangulated and derived categories follows below. Article references are usually to the first paper in this subject and may or may not be suitable as a source. Some projects may be too advanced. Some projects may need 2–3 persons. You are free to choose any topic, within or outside this list, as long as it at least touches on derived or triangulated categories.

You should give a 35 minute presentation (April 16–June 4) and also write a short introduction to the topic, ca 8 pages (due June 10). If there are several students choosing the same topic, please give a joint presentation (length gets multiplied with number of speakers) and submit a joint written report of a suitable length (again, roughly multiplied by number of authors).

Triangulated categories.

- (1) Tensor-triangulated geometry ([Bal05], ...)
- (2) Unbounded derived categories: K-injective, K-flats etc ([Spa88], Bökstedt–Neeman 1993, [Ser03], stacks project, ...)
- (3) Stable infinity categories (Lurie, ...)
- (4) t-structures and perverse sheaves ([BBD82]), ...
- (5) Classification of subcategories of $D^b(R)$ and $D(R)$: thick, localizing and smashing ([Nee92] and Hopkins 1985)
- (6) dg-categories

Derived categories.

- (7) Cotangent complexes (Lichtenbaum–Schlessinger 1967, Quillen 1970, André 1974, Illusie 1971, modern sources)
- (8) Dualizing complexes: perhaps also rigid ([Yek19, §13.1])
- (9) Grothendieck duality (very briefly in [Huy06, 3.4])
- (10) Spectral sequences and derived categories
- (11) Derived categories of quiver representations: also cluster algebras?
- (12) Derived categories of D-modules
- (13) Non-commutative deformations of derived categories

Derived equivalences (smooth projective varieties).

- (14) Reconstruction of X from $D(X)$ and description of $\text{Aut}(D^b(X))$ when X has an (anti-)ample canonical bundle. ([BO01], [Huy06, 4.1–4.2], this results dates back to 1995, see arXiv:alg-geom/9506012, Section 4)
- (15) Fourier–Mukai transforms and Orlov’s theorem ([Orl97, §2], [Orl03, Ch. 3], [Huy06, 5.1])
- (16) Abelian varieties and derived equivalence $D(A) \simeq D(\hat{A})$ ([Muk81], [Huy06, 9.1–9.2])
- (17) Derived McKay correspondence ([KV00])

Semi-orthogonal decompositions (smooth projective varieties).

- (18) Exceptional collections of line bundles on \mathbb{P}^n and toric/Fano varieties (Beilinson, ...)
- (19) Semi-orthogonal decompositions, projective bundle formula, blow-up formula (Bondal–Orlov, Orlov, ...)
- (20) Kuznetsov component (Kuznetsov, ...)
- (21) For algebraic stacks (Bergh, ...)

Moduli, Bridgeland stability conditions.

- (22) Moduli of perfect complexes (Lieblich)
- (23) Bridgeland stability condition (Bridgeland, ...)
- (24) Stable pairs and stability conditions (Le Potier, Pandharipande–Thomas, [Bay09])

Generation.

- (25) Classical and strong generators of $D^b(X)$ ([BvdB03])
- (26) Compact generators of $D(X)$ ([Nee96, BvdB03], effective version [LN07])

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