# MM3001 Final Exam Formula Sheet

## Quadratic equations

$ax^2 + bx + c, \quad a \neq 0$

$$x_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

when $b^2 - 4ac \geq 0$. Otherwise the equation has no roots.

## Change of base

Let $a$ a positive real number $a \neq 1$. Then

$$\log_a(b) = \frac{\ln(b)}{\ln(a)}$$

## Derivatives

| | |
|---|---|
| *Product rule* | $\frac{d}{dx}(f \cdot g)(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ |
| *Quotient rule* | $\frac{d}{dx}(f/g)(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$ |
| *Chain rule* | $\frac{d}{dx}(f \circ g)(x) = g'(x) f'(g(x))$ |
| *Elementary functions:* | $\frac{d}{dx} x^n = n x^{n-1}$<br>$\frac{d}{dx} e^x = e^x$<br>$\frac{d}{dx} \ln(x) = \frac{1}{x}$ |

## Taylor Formula

The Taylor polynomial of $f(x)$ degree $n$ around the point $x = a$ is given by the formula

$$p(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

The error is given by

$$|f(x) - p(x)| = \left| \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1} \right|$$

## Integration Methods

| | |
|---|---|
| *By parts* | $\int f(x)g(x)dx = F(x)g(x) - \int F(x)g'(x)dx$ |
| *By substitution* | $\int f(g(x))g'(x)dx = \int f(u)du$ where $u = g(x)$ |

## Primitives of elementary functions

- $n \neq -1$, $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$

## Geometric sums

$$\sum_{j=0}^{n} ar^n = a\frac{1 - r^{n+1}}{1 - r} = a\frac{r^{n+1} - 1}{r - 1}$$

## Critical points in 2 variables

Let $f(x, y)$ be a twice differentiable two-variate functions. The point $(x_0, y_0$ is a critical point if

$$f_x(x_0, y_0) = 0 = f_y(x_0, y_0).$$

The Hessian of $f(x, y)$ is

$$H(x, y) = \begin{vmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{vmatrix} = f_{xx}(x, y) f_{yy}(x, y) - f_{xy}(x, y) f_{yx}(x, y).$$

A critical point $(x_0, y_0)$ for $f$ is a:

1. local maximum point iff $H(x_0, y_0) > 0$ and $f_{xx}(x_0, y_0) < 0$ (and $f_{yy}(x_0, y_0) < 0$);
2. local minimum point iff $H(x_0, y_0) > 0$ and $f_{xx}(x_0, y_0) > 0$ (and $f_{yy}(x_0, y_0) > 0$);
3. saddle point iff $H(x_0, y_0) < 0$.