TEST 2 (Elementary Mathematics)

Students who now enter university or college courses in economics tend to have a wide range of mathematical backgrounds and aptitudes. At the low end, they may have no more than a shaky command of elementary algebra. Or, at the high end, they may already have a ready facility with calculus, though often it is some years since economics students took their last formal mathematics course. Experience suggests therefore that right from the start of the course, it is very important that the instructor, as well as each individual student, should get some impression of what the student knows well, what is vaguely familiar, and what seems to be more or less forgotten or perhaps never learned at all.

The present test is meant to test the students' actual knowledge of some elementary mathematical topics of interest to economists. The level is nevertheless more advanced than for Test 1 and the topics covered are discussed in the earlier chapters of the main text. The maximum total score is 100.

1. [Points: (a) 2; (b) 2; (c) 2; (d) 2]

(a) $2^5 + 2^5 = 2^x \implies x =$

(b) $3^{-15} + 3^{-15} + 3^{-15} = 3^y \implies y =$

(c) $\sqrt{13^2 - 5^2} =$

(d) $\dfrac{2^{26} - 2^{23}}{2^{26} + 2^{23}} = \dfrac{z}{9} \implies z =$

2. [Points: (a) 1+1; (b) 2+2]

(a) Find the slopes of the following straight lines:

(i) $y = -\dfrac{3}{2}x + 4$ (ii) $6x - 3y = 5$

(b) Find the equation of the straight line that:

(i) passes through $(-2, 3)$ and has slope -2; (ii) passes through both $(a, 0)$ and $(0, b)$.

3. [Points: 5] Fill in the following table and sketch the graph of $y = -x^2 + 2x + 4$.

x	-2	-1	0	1	2	3	4
$y = -x^2 + 2x + 4$							

4. [Points: 4+4] Determine the maximum/minimum points for:

(a) $y = x^2 - 4x + 8$ (b) $y = -2x^2 + 16x - 14$

5. [Points: 4+4] Perform the following polynomial divisions:

(a) $(2x^3 - 3x + 10) \div (x + 2)$ (b) $(x^4 + x) \div (x^2 - 1)$

6. [Points: 5] $f(x) = \frac{4}{3}x^3 - \frac{1}{5}x^5$. For what values of x is $f'(x) = 0$?

7. [Points: 3+3+3+3] Sketch the graph of a function f in each of the following cases:

(a) $f'(x) > 0$ and $f''(x) > 0$,

(b) $f'(x) > 0$ and $f''(x) < 0$

(c) $f'(x) < 0$ and $f''(x) > 0$,

(d) $f'(x) < 0$ and $f''(x) < 0$

8. [Points: 3+3]

(a) The cost in dollars of extracting T tons of a mineral ore is given by $C = f(T)$. Give an economic interpretation of the statement that $f'(1000) = 50$.

(b) A consumer wants to buy a certain item at the lowest possible price. Let $P(t)$ denote the lowest price found after searching the market for t hours. What are the likely signs of $P'(t)$ and $P''(t)$?

9. [Points: 2+2+2+2] Write down the general rules for differentiating the following:

(a) $y = f(x) + g(x) \Longrightarrow y' =$

(b) $y = f(x)g(x) \Longrightarrow y' =$

(c) $y = f(x)/g(x) \Longrightarrow y' =$

(d) $y = f(g(x)) \Longrightarrow y' =$

10. [Points: 2+2+2+2+2+2+2+2] Differentiate the following functions:

(a) $y = x^2$

(b) $y = x^5/5$

(c) $y = \dfrac{x}{x+1}$

(d) $y = (x^2+5)^6$

(e) $y = e^x$

(f) $y = \ln x$

(g) $y = 2^x$

(h) $y = x^x$

11. [Points: 2+2+2+2+2] Which of the following statements are correct?

(a) The rule that converts a temperature measured in degrees Fahrenheit into the same temperature measured in degrees Celsius is an invertible function.

(b) A concave function always has a maximum.

(c) A differentiable function can have an interior maximum only at a critical point for the function.

(d) If $f'(a) = 0$, then a is either a local maximum point or a local minimum point fo f.

(e) The conditions $f'(a) = 0$ and $f''(a) < 0$ are necessary and sufficient for a to be a local maximum point for f.

12. [Points: 2+2+2+2] In each of the following cases, decide whether the given formula is correct or not:

(a) $\displaystyle\int x^2\,\mathrm{d}x = \frac{1}{3}x^3 + C$

(b) $\displaystyle\int [f(x)+g(x)]\,\mathrm{d}x = \int f(x)\,\mathrm{d}x + \int g(x)\,\mathrm{d}x$

(c) $\displaystyle\int_a^b x\,\mathrm{d}x = b - a$

(d) $\displaystyle\int f(x)g(x)\,\mathrm{d}x = \int f(x)\,\mathrm{d}x \int g(x)\,\mathrm{d}x$