Lecture 2 solution to slides

- $[-2, 3) \cap (-1, 100] = (-1, 3)$
- $(-1, 1)^c = (-\infty, -1] \cup [1, +\infty)$
- $(-1, 7) \cup (-3, 5) = (-3, 7)$
- $|-100| = 100 \qquad |1000| = 1000 \qquad |\pi| = \pi$
- Find all x such that $|2x-6| < 1$

Case 1 $\quad 2x - 6 \geq 0 \quad$ (that is $x \geq 3$)

$$1 > |2x-6| = 2x-6$$

$$\iff \quad 7 > 2x \qquad x > \frac{7}{2} \quad \Rightarrow x \in [3, 7/2)$$

Case 2 $\quad 2x - 6 < 0 \quad$ (that is $x < 3$)

$$1 > |2x-6| = 6-2x$$

$$-5 > -2x \quad \Rightarrow \quad x > \frac{5}{2} \quad \Rightarrow \quad (5/2, 3)$$

The final solution is the UNION of the two cases

$$x \in (5/2, 3) \cup [3, 7/2) = (5/2, 7/2)$$

Find the natural domain of the following.

- $f(x) = \dfrac{1}{\sqrt{1-x} - 2}$

$1 - x \geq 0$ so $x \leq 1$ AND

$\sqrt{1-x} - 2 \neq 0$

now we want to solve $\sqrt{1-x} = 2$

we square both sides

$(1-x) = 4 \qquad x = -3$

Since squaring can produce extra solutions we have to check that $x=-3$ is indeed a solution

$\sqrt{1-(-3)} - 2 = \sqrt{4} - 2 = 0$ ✓

Answer $D = \{x \in \mathbb{R} \mid x \leq 1 \text{ and } x \neq -3\}$

- $g(t) = \sqrt{1 - |t|}$

we need $1 - |t| \geq 0$ so $t \in [-1, 1]$

$f(x) = x^2 \qquad g(x) = 2x+3$

$f \circ g\,(x) = (2x+3)^2$

$g \circ f\,(x) = 2x^2+3$

$f \circ g\,(0) = 9 \quad \neq \quad g \circ f\,(0) = 3$

they are NOT the same function

ORDER MATTERS

Linear growth of employees in a company

Last year 100 employees

Today 122 ——

In 6 years ?

if $f(x) = ax+b$ is the line

through $(-1, 100)$ $(0, 122)$ then

we need $f(6)$

First we need to find a and b

$b = f(0) = 122$

$a =$ slope formula $\dfrac{122-100}{0-(-1)} = 22$

$f(x) = 22x + 122$

Answer $f(6) = 132 + 122 = 254$

Quadratic optimization

(a) Profits = revenue $(100 \cdot x)$
$\quad - \text{costs} \quad (20x + 0.25x^2)$

$$f(x) = 100x - 20x - 0.25x^2$$
$$= 80x - 0.25x^2$$

has a max in $x = -\dfrac{80}{-2 \cdot 0.25} = \dfrac{40}{0.25} = 160$

(b) with taxation the costs become

$$20x + 10x + 0.25x^2$$

So the profits are modeled by

$$f(x) = 70x - 0.25x^2$$

max in $x = 140$

(c) Profits : $f(x) = (p - \alpha - \tau)x - \beta x^2$

since $\beta > 0$ $-\beta < 0$ and this has a max in

$$x = -\frac{(p - \alpha - \tau)}{2(-\beta)} = \frac{(p - \alpha - \tau)}{2\beta}$$