Stockholm University

MM3001

Long Division of Polynomials

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Section 1
Long Division for Polynomials

Set up

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We want to divide a polynomial

$$p(x) = c_d x^d + c_{d-1} x^{d-1} + \cdots + c_1 x + c_0$$

by a non zero polynomial

$$q(x) = q_e x^e + q_{e-1} x^{e-1} + \cdots + q_0$$

Set up

We want to divide a polynomial

$$p(x) = c_d x^d + c_{d-1} x^{d-1} + \cdots + c_1 x + c_0$$

by a non zero polynomial

$$q(x) = q_e x^e + q_{e-1} x^{e-1} + \cdots + q_0$$

That is we want to find $k(x)$ and $r(x)$ such that

$$p(x) = k(x)q(x) + r(x)$$

We set up a table like the following

$$\begin{array}{c|c} & \\ \hline \text{Write } q(x) \text{ with powers} & p(x) \text{ with powers} \\ \text{in descending order} & \text{descending order and gap filled} \end{array}$$

Set up

We want to divide a polynomial

$$p(x) = c_d x^d + c_{d-1} x^{d-1} + \cdots + c_1 x + c_0$$

by a non zero polynomial

$$q(x) = q_e x^e + q_{e-1} x^{e-1} + \cdots + q_0$$

That is we want to find $k(x)$ and $r(x)$ such that

$$p(x) = k(x)q(x) + r(x)$$

We set up a table like the following

	Here will be the result
Write $q(x)$ with powers in descending order	$p(x)$ with powers descending order and gap filled

Example

As we did in class we take $p(x) = x^3 - 11x + 6$ and $q(x) = x - 3$.

$$
\begin{array}{r|llll}
 & & & & \\
\hline
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & & & & \\
 & & & & \\
\end{array}
$$

Step 1:

Divide the leading term of $p(x)$ by the leading term of $q(x)$ write the result in the first free space over the line.
The leading term of $x^3 - 11x + 6$ is x^3. The leading term of $x - 3$ is x.
We have

$$\frac{x^3}{x} = x^2$$

So we write

$$\begin{array}{r|llll} & & x^2 & & \\ \cline{2-5} x-3 & x^3 & +0x^2 & -11x & +6 \\ & & & & \\ & & & & \end{array}$$

Step 2

Multiply what you got by $-q(x)$ and write the result below $p(x)$. Be careful to keep the powers in the right columns! In our case we have

$$x^2(-(x-3)) = -x^3 + 3x^2$$

So we get

$$\begin{array}{r|llll} & & x^2 & & \\ \hline x-3 & x^3 & +0x^2 & -11x & +6 \\ & -x^3 & +3x^2 & & \end{array}$$

Step 3

Sum the two polynomials and write the result under a line In our case we have

$$
\begin{array}{r|rrrr}
 & & x^2 & & \\
\hline
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & -x^3 & +3x^2 & & \\
\hline
 & // & 3x^2 & -11x & +6 \\
 & & & & \\
\end{array}
$$

Step 4

Repeat the previous steps using the new polynomial in the role of $p(x)$. You can stop when the polynomial you get after summing has a lower degree than $q(x)$.

Final Answer What is written above the line on the right is your $k(x)$, the polynomial you got after the last sum is the remainder $r(x)$

Example

$$
\begin{array}{r|rrrr}
 & & x^2 & +3x & \\
\hline
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & -x^3 & +3x^2 & & \\
\hline
 & // & 3x^2 & -11x & +6 \\
\end{array}
$$

Example

$$
\begin{array}{r|rrrr}
 & & x^2 & +3x & \\
\cline{2-5}
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & -x^3 & +3x^2 & & \\
\cline{2-5}
 & // & 3x^2 & -11x & +6 \\
 & & -3x^2 & +9x & \\
\cline{3-5}
 & & & & \\
\cline{4-5}
 & & & &
\end{array}
$$

Example

$$
\begin{array}{r|rrrr}
 & & x^2 & +3x & \\
\cline{2-5}
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & -x^3 & +3x^2 & & \\
\cline{2-5}
 & // & 3x^2 & -11x & +6 \\
 & & -3x^2 & +9x & \\
\cline{3-5}
 & & // & -2x & +6 \\
 & & & & \\
\cline{4-5}
 & & & &
\end{array}
$$

Example

$$
\begin{array}{r|rrrr}
 & & x^2 & +3x & -2 \\
\cline{2-5}
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & -x^3 & +3x^2 & & \\
\cline{2-5}
 & // & 3x^2 & -11x & +6 \\
 & & -3x^2 & +9x & \\
\cline{3-5}
 & & // & -2x & +6 \\
 & & & & \\
\cline{4-5}
 & & & &
\end{array}
$$

Example

$$
\begin{array}{r|rrrr}
 & & x^2 & +3x & -2 \\
\hline
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & -x^3 & +3x^2 & & \\
\hline
 & // & 3x^2 & -11x & +6 \\
 & & -3x^2 & +9x & \\
\cline{3-5}
 & & // & -2x & +6 \\
 & & & 2x & -6 \\
\cline{4-5}
 & & & & \\
\end{array}
$$

Example

$$
\begin{array}{r|rrrr}
 & & x^2 & +3x & -2 \\
\cline{2-5}
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & -x^3 & +3x^2 & & \\
\cline{2-5}
 & // & 3x^2 & -11x & +6 \\
 & & -3x^2 & +9x & \\
\cline{3-5}
 & & // & -2x & +6 \\
 & & & 2x & -6 \\
\cline{4-5}
 & & & // & //
\end{array}
$$

Example

$$
\begin{array}{r|rrrr}
 & & x^2 & +3x & -2 \\ \cline{2-5}
x-3 & x^3 & +0x^2 & -11x & +6 \\
 & -x^3 & +3x^2 & & \\ \cline{2-5}
 & // & 3x^2 & -11x & +6 \\
 & & -3x^2 & +9x & \\ \cline{3-5}
 & & // & -2x & +6 \\
 & & & 2x & -6 \\ \cline{4-5}
 & & & // & //
\end{array}
$$

Answer: We have that $k(x) = x^2 + 3x - 2$ and $r(x) = 0$.

Another example

$$
\begin{array}{r|rrrrrrrr}
 & & & & x^4 & +x^3 & -x^2 & -x & \\
\cline{2-9}
x^3+x+1 & x^7 & +x^6 & & +x^4 & & +x^2 & +x & +1 \\
 & -x^7 & & -x^5 & -x^4 & & & & \\
\cline{2-9}
 & & x^6 & -x^5 & & & +x^2 & +x & +1 \\
 & & -x^6 & & -x^4 & -x^3 & & & \\
\cline{3-9}
 & & & -x^5 & -x^4 & -x^3 & +x^2 & +x & +1 \\
 & & & x^5 & & +x^3 & +x^2 & & \\
\cline{4-9}
 & & & & -x^4 & & +2x^2 & +x & +1 \\
 & & & & x^4 & & x^2 & x & \\
\cline{5-9}
 & & & & & & 3x^2 & +2x & +1
\end{array}
$$

Answer: We have that $k(x) = x^4 + x^3 - x^2 - x$ and the remainder is $r(x) = 3x^2 + 2x + 1$

Ruffini's trick

Again we want to divide a polynomial

$$p(x) = c_d x^d + c_{d-1} x^{d-1} + \cdots + c_1 x + c_0$$

by another polynomial $q(x)$ When $q(x) = x + a$, then there is a quick method to do the long division. Start with writing the coefficients in a table like that

$$\begin{array}{r|cccc|c} & c_d & c_{d-1} & \cdots & c_1 & c_0 \\ -a & & & & & \\ \hline & c_d & & & & \end{array}$$

Ruffini's trick

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$$p(x) = c_d x^d + c_{d-1} x^{d-1} + \cdots + c_1 x + c_0$$

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$$\begin{array}{r|cccc|c} & c_d & c_{d-1} & \cdots & c_1 & c_0 \\ -a & & & & & \\ \hline & c_d & & & & \end{array}$$

Attention: Note the sign in front of a

In the example

Again take $p(x) = x^3 - 11x + 6$ and $q(x) = x - 3$ The table becomes

$$\begin{array}{r|ccc|c} & 1 & 0 & -11 & +6 \\ 3 & & & & \\ \hline & 1 & & & \end{array}$$

The step

Multiply the rightmost element below the line by $-a$ write the result above the line in the next column.

$$
\begin{array}{c|ccc|c}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & & \\
\hline
 & 1 & & &
\end{array}
$$

The step

Multiply the rightmost element below the line by $-a$ write the result above the line in the next column.

$$
\begin{array}{r|rrr|r}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & & \\
\hline
 & 1 & & &
\end{array}
$$

Make the sum of the numbers appearing in the column and write it under the line

$$
\begin{array}{r|rrr|r}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & & \\
\hline
 & 1 & 3 & &
\end{array}
$$

The step

Multiply the rightmost element below the line by $-a$ write the result above the line in the next column.

$$\begin{array}{c|ccc|c} & 1 & 0 & -11 & +6 \\ 3 & & 3 & & \\ \hline & 1 & & & \end{array}$$

Make the sum of the numbers appearing in the column and write it under the line

$$\begin{array}{c|ccc|c} & 1 & 0 & -11 & +6 \\ 3 & & 3 & & \\ \hline & 1 & 3 & & \end{array}$$

Reiterate

$$\begin{array}{c|ccc|c} & 1 & 0 & -11 & +6 \\ 3 & & 3 & 9 & \\ \hline & 1 & 3 & & \end{array}$$

The step

Multiply the rightmost element below the line by $-a$ write the result above the line in the next column.

$$
\begin{array}{c|ccc|c}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & & \\
\hline
 & 1 & & &
\end{array}
$$

Make the sum of the numbers appearing in the column and write it under the line

$$
\begin{array}{c|ccc|c}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & & \\
\hline
 & 1 & 3 & &
\end{array}
$$

Reiterate

$$
\begin{array}{c|ccc|c}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & 9 & \\
\hline
 & 1 & 3 & -2 &
\end{array}
$$

On the bottom line between the two vertical lines you find the coefficents of k. On the bottom right corner you find the remainder.

The step

Multiply the rightmost element below the line by $-a$ write the result above the line in the next column.

$$\begin{array}{r|rrr|r} & 1 & 0 & -11 & +6 \\ 3 & & 3 & & \\ \hline & 1 & & & \end{array}$$

Make the sum of the numbers appearing in the column and write it under the line

$$\begin{array}{r|rrr|r} & 1 & 0 & -11 & +6 \\ 3 & & 3 & & \\ \hline & 1 & 3 & & \end{array}$$

Reiterate

$$\begin{array}{r|rrr|r} & 1 & 0 & -11 & +6 \\ 3 & & 3 & 9 & -6 \\ \hline & 1 & 3 & -2 & \end{array}$$

On the bottom line between the two vertical lines you find the coefficents of k. On the bottom right corner you find the remainder.

The step

Multiply the rightmost element below the line by $-a$ write the result above the line in the next column.

$$
\begin{array}{r|rrr|r}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & & \\
\hline
 & 1 & & & \\
\end{array}
$$

Make the sum of the numbers appearing in the column and write it under the line

$$
\begin{array}{r|rrr|r}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & & \\
\hline
 & 1 & 3 & & \\
\end{array}
$$

Reiterate

$$
\begin{array}{r|rrr|r}
 & 1 & 0 & -11 & +6 \\
3 & & 3 & 9 & -6 \\
\hline
 & 1 & 3 & -2 & 0 \\
\end{array}
$$

On the bottom line between the two vertical lines you find the coefficents of k. On the bottom right corner you find the remainder.

Conclusion

$$\begin{array}{r|rrr|r} & 1 & 0 & -11 & +6 \\ 3 & & 3 & 9 & -6 \\ \hline & 1 & 3 & -2 & 0 \end{array}$$

On the bottom line between the two vertical lines you find the coefficents of k. On the bottom right corner you find the remainder. Thus $k(x) = x^2 + 3x - 2$ and $r(x) = 0$.

Another example

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Let $p(x) = x^6 + 2x^4 + x + 3$ and $q(x) = x + 2$

$$\begin{array}{c|cccccc|c} & 1 & 0 & 2 & 0 & 0 & 1 & 3 \\ -2 & & -2 & 6 & -12 & +24 & -48 & 94 \\ \hline & 1 & -2 & 4 & -12 & 24 & -47 & 97 \end{array}$$

The answer is $k(x) = x^5 - 2x^4 - 12x^3 + 24x - 47$ and the remainder is $r(x) = 97$.

Thank you for your attention!