Lecture 3 Solution to slide exercises

$f(x) = Ca^x$ find C and a such that

$f(-1) = \frac{2}{3}$ $\quad f(3) = 54$

If $C = 0$ then $f(-1) = 0 \neq 2/3$ thus we can assume that $\boxed{C \neq 0}$

$$f(3)/f(-1) = 54 \cdot \frac{3}{2} = 81$$

$$\frac{Ca^3}{Ca^{-1}} = a^4$$

So a can be only 3 or -3, however we know that $a > 0$ (since it is the base of an exponential function. So

$$\boxed{a = 3}$$

Now $\quad \frac{2}{3} = Ca^{-1} = C/3$

Thus $\quad \boxed{C = 2}$

And $\quad f(x) = 2 \cdot 3^x$

$$\frac{\ln\left(2^{3/8}\right)}{\ln\left(2^{-1/2}\right)} = \frac{3/8}{-1/2} \quad \frac{\ln(2)}{\ln(2)} = -\frac{3}{4} \cdot 2 = -\frac{3}{4}$$

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8. $100 = 200\,(0.8)^t$

$$(0.8)^t = \frac{1}{2}$$

$$t \cdot \ln(0.8) = \ln(1/2)$$

$$t = \ln(1/2) \,/\, \ln(0.8)$$

$$K(t) = K_0\left(1 + \frac{2}{100}\right)^t = 2K_0$$

$$\left(1 + \frac{2}{100}\right)^t = 2$$

$$t \ln(1.02) = \ln(2)$$

$$t = \frac{\ln(2)}{\ln(1.02)}$$