

Lecture 4 (Geometric Series) slide solution

1) \$5000 invested 6% a year compounded monthly. How long to have \$6000

monthly interest rate $\frac{6}{12}\% = 0.5\%$

$$5000(1.005)^t = 6000$$

$$t = \ln(6/5) / \ln(1.005) \approx 36.5 \text{ month}$$

3 years and a month

2) We have to see how much we will have in the account in 10 years

OPTION 1: cash

We give all the money and we are left with 0. In 10 years we will have 0

OPTION 2

in 10 years we will have

$$\$13000 \left(1 + \frac{2}{12 \cdot 100}\right)^{120} \text{ in the bank}$$

but we will owe

$$\$13000 \left(1 + \frac{6}{12 \cdot 100}\right)^{120} \text{ which is a bigger number}$$

So this is not a convenient solution

OPTION 3:

We have $13000 - 2000 = 11000$ now

In one year (1st payment)

$$11000(1.02) - 1300$$

In two years (2nd payment)

$$11000(1.02)^2 - 1300(1.02) - 1300$$

In 9 years (Last payment)

$$\begin{aligned} & 11000(1.02)^9 - \sum_{i=0}^8 1300(1.02)^i \\ &= 11000(1.02)^9 - \frac{(1.02)^9 - 1}{0.2} \cdot 1300 \\ &= 11000(1.02)^9 - \left((1.02)^9 - 1 \right) \cdot 1300 \cdot \frac{100}{2} \\ &= 11000(1.02)^9 - \left((1.02)^9 - 1 \right) 65000 \\ &= 65000 - (65000 - 11000)(1.02)^9 \\ &= 65000 - (54000)(1.02)^9 \\ &= \$465 \end{aligned}$$

$$\text{In 10 years} = 465(1.02) > 0$$

Company buys a building site:

Also here we have to evaluate the 3 options after 12 years:

OPTION 1 0\$ in 12 years

OPTION 2

$$670000(1.115)^8 - \sum_{i=0}^8 120000(1.115)^i$$
$$= 670000(1.115)^8 - 120000 \frac{(1.115)^9 - 1}{0.115} < 0$$

In 8 years the money on the bank account will be finished.

OPTION 3

$$(450000)(1.115)^{12} - \sum_{i=0}^{11} 70000(1.115)^i$$
$$= 450000(1.115)^{12} - 70000 \frac{(1.115)^{12} - 1}{0.115} \hat{=} 22741 > 0$$

So the third option is the most convenient

The trust funds

20 payments

$$K(1.035)^{19} - \sum_{i=2}^{19} 1000(1.035)^i = 50000$$

$$K = \frac{1}{(1.035)^9} \left[1000 \frac{(1.035)^{10} - 1}{0.035} \right] + 50000$$

$$\approx 40716.62$$