

Lecture 5 slide solution

$$\bullet \lim_{x \rightarrow 2} \frac{x^2 - 3x + 1}{x - 1} = -1$$

(just plug in 2)

$$\bullet \lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1}$$

if we plug in $x=1$ we get $\frac{0}{0}$ indeterminate

$$\stackrel{=}{=} \lim_{x \rightarrow 1} \frac{(x-1)(x-2)}{x-1} = -1$$

$$\bullet \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$$

if we plug $x=0$ we have $\frac{0}{0}$ indeterminate

$$\stackrel{=}{=} \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} \cdot \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} =$$

$$\stackrel{=}{=} \lim_{x \rightarrow 0} \frac{x+1-1}{x} = \lim_{x \rightarrow 0} \frac{x}{x} = 1$$

$$\bullet \lim_{x \rightarrow 1} \frac{x^2 - 3x + 1}{x - 1}$$

if we plug in $x=1$ we get $\frac{-1}{0}$

but if we approach 0 from the left (negative $\neq$) we get bigger and bigger $\neq$ if we approach 0 from the right we get smaller and smaller numbers

$\Rightarrow$ The limit does not exist.

• Find the line tangent to $y = x^2 + 2x - 1$ in $P = (2, 7)$

Notice that if we plug in $x = 2$ in the equation we have that $y = 7$ so $(2, 7)$ belongs to the curve.

$$y' = 2x + 2 \quad \text{we plug in } x = 2$$

$$y'(2) = 4 + 2 = 6 \quad \text{this is the slope of the tg.}$$

Now we know that the line goes through $(2, 7)$ so the equation looks like

$$y = 6x + m \quad \text{and} \quad 7 = 6 \cdot 2 + m$$

$$m = 7 - 12 = -5$$

Answer

$$\boxed{y = 6x - 5}$$

The initial problem

$$\bullet S(Y) = S_0 + S \cdot Y$$

$$S'(Y) = S$$

$$\bullet S(Y) = 100 + 0.1Y + 0.0002Y^2$$

$$S'(Y) = 0.1 + 0.0004Y$$

Examples

$$\bullet a^x = \left(e^{\ln(a)} \right)^x = e^{\ln(a)x}$$

$$\begin{aligned} \frac{d}{dx} a^x &= \ln(a) e^{\ln(a)x} \\ &= \ln(a) a^x \end{aligned}$$

$$\bullet \log_a(x) = \frac{\ln(x)}{\ln(a)}$$

$$\frac{d}{dx} \log_a(x) = \frac{1}{\ln(a)} \cdot \frac{1}{x}$$

$$\bullet \sqrt{e^{\sqrt{x^2+1}}} = \left(e^{\sqrt{x^2+1}} \right)^{\frac{1}{2}} = e^{\frac{1}{2}\sqrt{x^2+1}}$$

$$\frac{d}{dx} \sqrt{e^{\sqrt{x^2+1}}} = \frac{d}{dx} \left(\frac{1}{2} \sqrt{x^2+1} \right) \cdot e^{\frac{1}{2}\sqrt{x^2+1}}$$

$$= \frac{2x}{2} \cdot \frac{1}{\sqrt{x^2+1}} \cdot e^{\frac{1}{2}\sqrt{x^2+1}}$$

$$= \frac{x}{\sqrt{x^2+1}} \cdot e^{\frac{1}{2}\sqrt{x^2+1}}$$

• $\ln|x|$ has domain $x \neq 0$

• $x > 0$ $\ln|x| = \ln(x)$

$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$

• $x < 0$ $\ln|x| = \ln(-x)$

$$\begin{aligned} \frac{d}{dx} \ln|x| &= \frac{d}{dx} (-x) \cdot \left(\frac{d}{dx} \ln \right) (-x) \\ &= -1 \cdot \frac{1}{-x} = \frac{1}{x} \end{aligned}$$

Thus $\frac{d}{dx} \ln|x| = \frac{1}{x}$ when $x \neq 0$

• $\frac{d}{dx} \left[(2x-1) (e^{x^2}-2x+1)^{25} \right]$

$$= 2(e^{x^2}-2x+1)^{25} + (2x-1)(2xe^{x^2}-2) \cdot 25 \cdot (e^{x^2}-2x+1)^{24}$$

$$= (e^{x^2} - 2x + 1)^{24} \cdot \left(2(e^{x^2} - 2x + 1) + 4x^2 e^{x^2} - 4x - 2xe^{x^2} + 2 \right)$$

$$= (e^{x^2} - 2x + 1)^{24} \left(2e^{x^2} - 8x + 4 + 4x^2 e^{x^2} - 2xe^{x^2} \right)$$

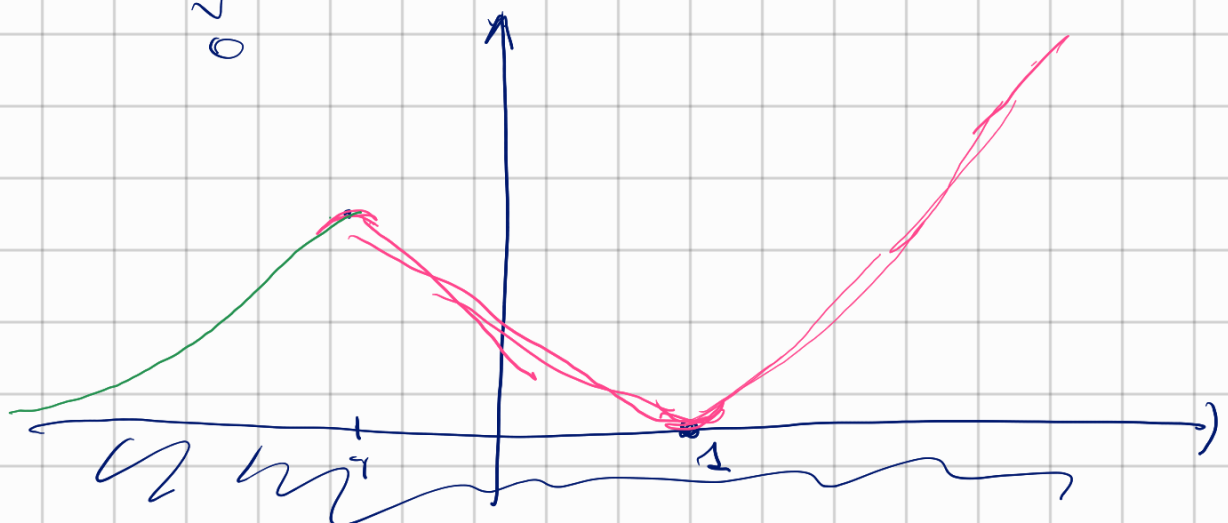
$$\bullet f(x) = e^x (x^2 - 2x + 1) = e^x (x-1)^2 \geq 0$$

$$f'(x) = e^x (x^2 - 2x + 1) + e^x 2(x-1)$$

$$= e^x (x-1)^2 + e^x 2(x-1)$$

$$= e^x (x-1) (x-1+2)$$

$$= e^x (x-1) (x+1)$$



$f'(x) > 0$ if and only if

$$(x-1)(x+1) > 0$$

$$\parallel$$
$$x^2 - 1$$



Thus $f(x)$ is increasing
whenever $x < -1$ and $x > 1$

$$f(-1) =$$