Lecture 6 exercise solution

- $$a - b(P+\tau) = \alpha + \beta P$$

$P=P(\tau)$ we take derivative

$$-b(P'(\tau)+1) = \beta P'(\tau)$$

solve for $P'(\tau)$

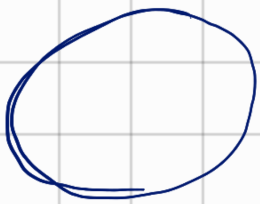
$$(\beta+b)P'(\tau) = -b$$

$$P'(\tau) = -\frac{b}{\beta+b} < 0$$

which means that the equilibrium price goes down if the tax goes up.

- Slope of the tangent at the unit circle

$$y^2 + x^2 = 1$$



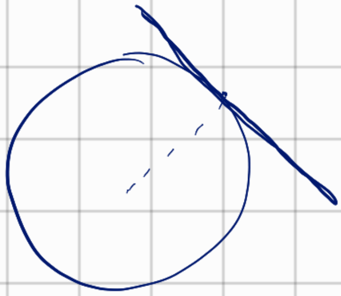
if $y \neq 0$ ($x \neq \pm 1$) we can consider locally $y = y(x)$ a function of x

We derive both sides

$$2y'(x)y(x) + 2x = 0$$

$$y'(x) = \frac{-2x}{2\cdot y(x)} \qquad (x \neq \pm 1 \;\; y(x) \neq 0)$$

$$= \frac{-x}{y}$$



in $\left(\frac{\sqrt{2}}{2} \; \frac{\sqrt{2}}{2}\right)$ es -1

- $y^3 e^x + x^2 e^y + y^3 - 2y = C$

We plug in $(x y) = (1,1)$

$$1^3 e^1 + 1^2 \cdot e^1 + 1^3 - 2 = C$$

$$C = 2e - 1$$

Now we consider y as a function of x that is

$$y = y(x)$$

and we derivate both sides

$$y'(x)\cdot 3y(x)^2 e^x + y(x)^3 e^x + 2xe^y + y'(x)x^2 e^y$$
$$+ 3y'(x)y(x)^2 - 2y'(x) = 0$$

Now we impose $x=1$ $y(1)=1$

$$y'(1)\cdot 3\cdot 1^2\cdot e + 1^3 e + 2\cdot 1\cdot e + y'(1)1^2 e$$
$$+ 3y'(1)\cdot 1^2 - 2y'(1) = 0$$

SOLVE FOR $y'(1)$

$$y'(1)\left(4e+1\right) = -3e$$

$$\boxed{y'(1) = -\frac{3e}{4e+1}}$$

The tangent line has equation

$$y = y'(1)x + m$$
$$= -\frac{3e}{4e+1}x + m$$

Since it passes throug $(1,1)$ we have

$$1 = -\frac{3e}{4e+1} + m$$

$$m = 1 + \frac{3e}{4e+1} = \frac{4e+1+3e}{4e+1}$$
$$= \frac{7e+1}{4e+1}$$

So the equation is

$$y = -\frac{3e}{4e+1}x + \frac{7e+1}{4e+1}$$

- $$y^2x^2 + \frac{x}{\sqrt{y}} = 6$$

we consider $y = y(x)$ and derivate on both sides

$$2y'(x)\,y(x)\cdot x^2 + y(x)^2\cdot 2x + \frac{1}{\sqrt{y}} - \frac{1}{2}x y'(x)\left(y(x)\right)^{-3/2} = 0 \quad (*)$$

Now we impose $x=2$ $y(2)=1$

$$2y'(2)\cdot 1\cdot 4 + 1^2\cdot 4 + 1 - y'(2) = 0$$

Solve for $y'(2)$

$$8y'(2) + 4 + 1 - y'(2) = 0$$

$$7y'(2) = -5$$

$$\boxed{y'(2) = -\frac{5}{7}}$$

For the second derivative we have to implicitly derive (*)

$$2y''(x)\,y(x)\cdot x^2 + 2y'(x)\cdot y'(x)\cdot x^2$$

$$+ 4y'(x)\,y(x)\cdot x \quad + 4y'(x)\,y(x)\cdot x$$

$$+ 2y(x)^2 - \frac{1}{2}\,y'(x)\left(y(x)\right)^{-3/2}$$

$$-\frac{1}{2}\,y'(x)\left(y(x)\right)^{-3/2} \quad -\frac{1}{2}\,x\,y''(x)\left(y(x)\right)^{-3/2}$$

$$+\frac{1}{4}\,x\,y'(x)\cdot y'(x)\left(y(x)\right)^{-5/2} = 0$$

Now we set $x=2 \quad y(2)=1 \quad y'(2) = -\frac{5}{7}$

$$8y''(2) + \frac{50}{49}\cdot 4 \quad -\frac{40}{7} \quad -\frac{40}{7} + 2 + \frac{5}{14} + \frac{5}{14}$$

$$-1\cdot y''(2) + \frac{1}{2}\,\frac{25}{49} \quad = 0$$

Solve for $y''(2)$

$$7y''(2) + \frac{400 - 1120 + 196 + 70 + 25}{98} = 0$$

$$7\,y''(2) = \frac{429}{98}$$

$$y''(2) = \frac{429}{668}$$