# Exam question

$$f(x) = (x^2-9)e^{5x}$$

Domain: $\mathbb{R}$

Zeroes: $f(x)=0 \iff x^2-9=0$

$$x = \pm 3$$

<u>Critical points</u> $\quad f'(x)=0$

$$f'(x) = 2x e^{5x} + (x^2-9)5e^{5x}$$

$$= e^{5x}(5x^2+2x-45) = 0$$

$$\iff 5x^2+2x-45=0$$

$$x = \frac{-1\pm\sqrt{226}}{5}$$

we have two critical points

$$\frac{-1-\sqrt{226}}{5} \approx -3.2 \quad \& \quad \frac{-1+\sqrt{226}}{5} \approx 2.8$$

| | | $\frac{-1-\sqrt{226}}{5}$ | | $\frac{-1+\sqrt{226}}{5}$ | |
|---|---|---|---|---|---|
| $f'(x)$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $f(x)$ | $\nearrow$ | $-$ | $\searrow$ | $-$ | $\nearrow$ |
| | | local max | | local min. | |

$$f''(x) = 5e^{5x}(5x^2+2x-45) + e^{5x}(10x+2)$$

$$= e^{5x}(25x^2+10x-225+10x+2)$$

$$= e^{5x}(25x^2+20x-223) = 0$$

$$x = \frac{-10 \pm \sqrt{100+223\cdot 25}}{25}$$

$$= \frac{-10 \pm \sqrt{5675}}{25} \approx -3.41,\ 2.61$$

The function is convex for

$$x \leq \frac{-10-\sqrt{5675}}{25} \quad \text{and} \quad x > \frac{-10+\sqrt{5675}}{25}$$

and is concave for

$$\frac{-10-\sqrt{5675}}{25} < x < \frac{-10+\sqrt{5675}}{25}$$

<u>Note</u> $f(-3.2) = 1.39$

$$f(2.8) \simeq \underline{-13950200.96963}$$

Geogebra has a scaling problem here

try and put x and y in scale $1:10^6$ and you will see the min.

# Limits

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} (x^2-9)\, e^{5x}$$

$$= +\infty$$

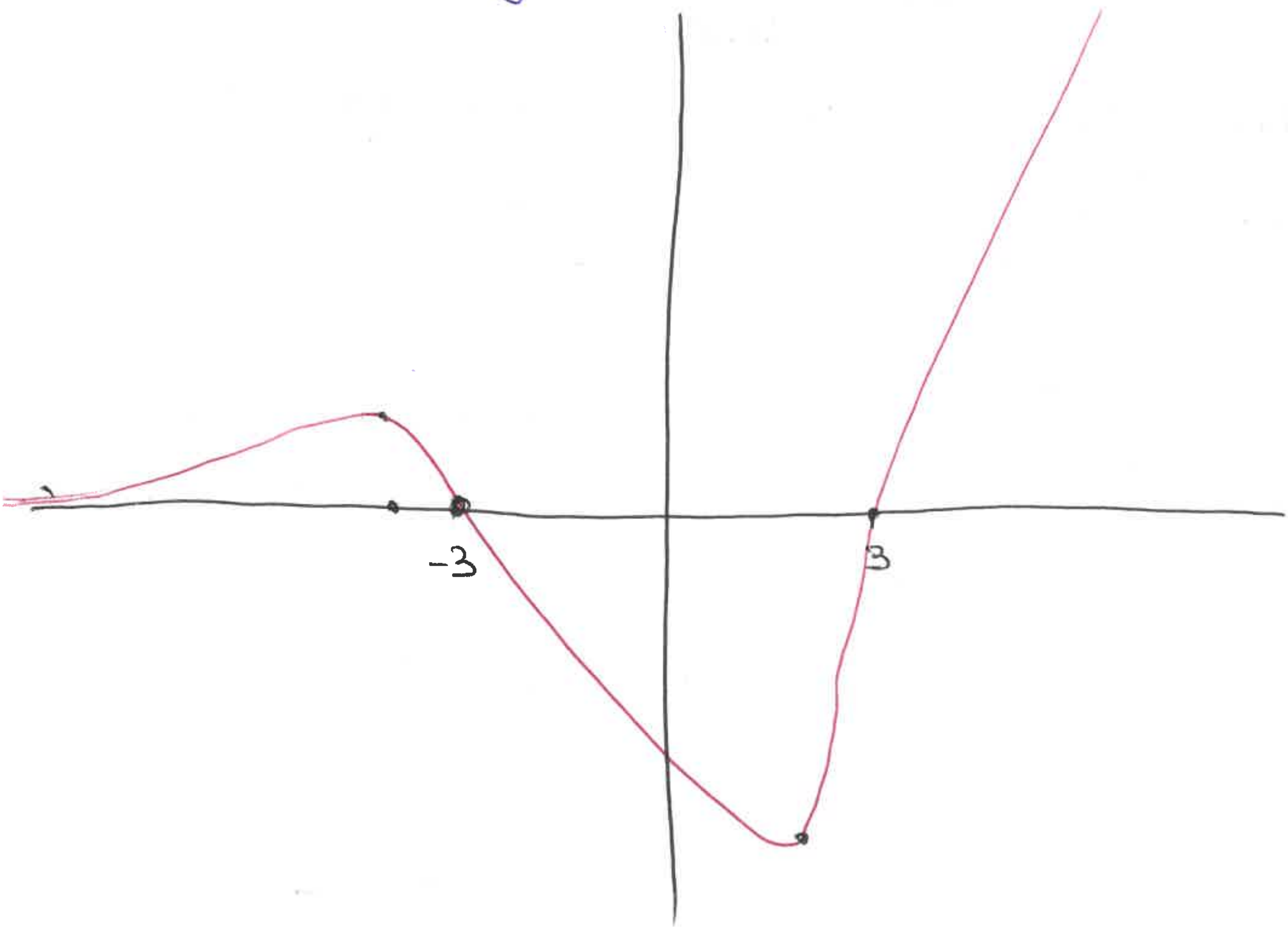
$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (x^2-9)e^{5x} = +\infty \cdot 0$$

indetermined form

$$= \lim_{x \to -\infty} \frac{x^2-9}{e^{-5x}} = \frac{+\infty}{+\infty}$$ I can use Hôpital

$$= \lim_{x \to -\infty} \frac{2x}{-5e^{-5x}} \overset{H}{=} \lim_{x \to -\infty} \frac{2}{e^{-5x}} = 0$$

Sketch of the graph (not in scale)



Exam question

$$f(x) = x^3(x-1) = x^4 - x^3$$

$$f'(x) = 4x^3 - 3x^2 = x^2(4x-3)$$

$f'(x) \geq 0$ if $4x-3 \geq 0$ $\quad 4x \geq 3 \quad x \geq \frac{3}{4}$

$f'(x) = 0$ if $x = 0$ & if $x = \frac{3}{4}$

| | | $0$ | | $\frac{3}{4}$ | |
|---|---|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $-$ | $0$ | $+$ |
| $f(x)$ | $\searrow$ | | $\searrow$ | | $\nearrow$ |
| | | inflection | | local min | |

we have that $f(x)$ is increasing when $x \geq \frac{3}{4}$ and decreasing when $x \leq \frac{3}{4}$

We want max & min in the interval $[-1, 1]$

we have two critical points $0, \frac{3}{4}$ & both of them lie in the interval

Candidates for max & min points

$-1, 1, 0, \frac{3}{4}$

Min.

$\boxed{f(-1) = 2}$ $\quad f(1) = 0 \quad f(0) = 0 \quad \boxed{f\left(\frac{3}{4}\right) = -\frac{3^3}{4^4}} < 0$

max

- we have ~~toso~~ that the function has max value 2 taken at the point $x=-1$

- the function on $[-1,1]$ has min value $\frac{-3^3}{4^4}$ taken at the point $x=\frac{3}{4}$

Complute $\lim\limits_{x\to+\infty} f(x)/e^x$

$$= \lim_{x\to+\infty} \frac{x^4-x^3}{e^x} =$$

$$= \lim_{x\to+\infty} \frac{x^4}{e^x} - \frac{x^3}{e^x}$$

we have seen in class that both summand tend to 0

$$= \boxed{0}$$

Alternatively use Hôpital 4 times

$$= \lim_{x\to+\infty} \frac{24}{e^x} = 0$$