Lecture 8 solution to slides

- $Q(L) = 12L^2 - \frac{1}{20} L^3$

$$Q'(L) = 24L - \frac{3}{20} L^2$$

$$= L\left(24 - \frac{3}{20} L\right)$$

$L = 0$ and $L = \frac{20 \cdot 24}{3} = 160$

Candidates

$L = 0$ $\quad Q(L) = 0$

$L = 200$ $\quad Q(200) = 80000$

$L = 160$ $\quad \boxed{Q(160) = 102400}$

max value.

$$f(x) = \sqrt{x}\, e^{-x}$$

Domain $x \geq 0$

$$f'(x) = \frac{1}{2\sqrt{x}} e^{-x} - \sqrt{x}\, e^{-x}$$

$$= \underset{0<}{e^{-x}} \left(\frac{1}{2\sqrt{x}} - \sqrt{x} \right) \geq 0$$

$\boxed{x \neq 0}$

when $\dfrac{1}{2\sqrt{x}} - \sqrt{x} \geq 0$

$$\frac{1}{2\sqrt{x}} \geq \sqrt{x} \qquad (\sqrt{x} > 0)$$

$$\frac{1}{2} \geq \sqrt{x} \cdot \sqrt{x} = x$$

Thus we have that $f'(x) > 0$ when $x > \frac{1}{2}$ and $f'(\frac{1}{2}) = 0$

Thus f is increasing for $x \leq \frac{1}{2}$ and decreasing for $x \geq \frac{1}{2}$

$$
\begin{array}{cccc}
 & & \frac{1}{2} & \\
f'(x) & + & 0 & - \\
f(x) & \nearrow & - & \searrow
\end{array}
$$

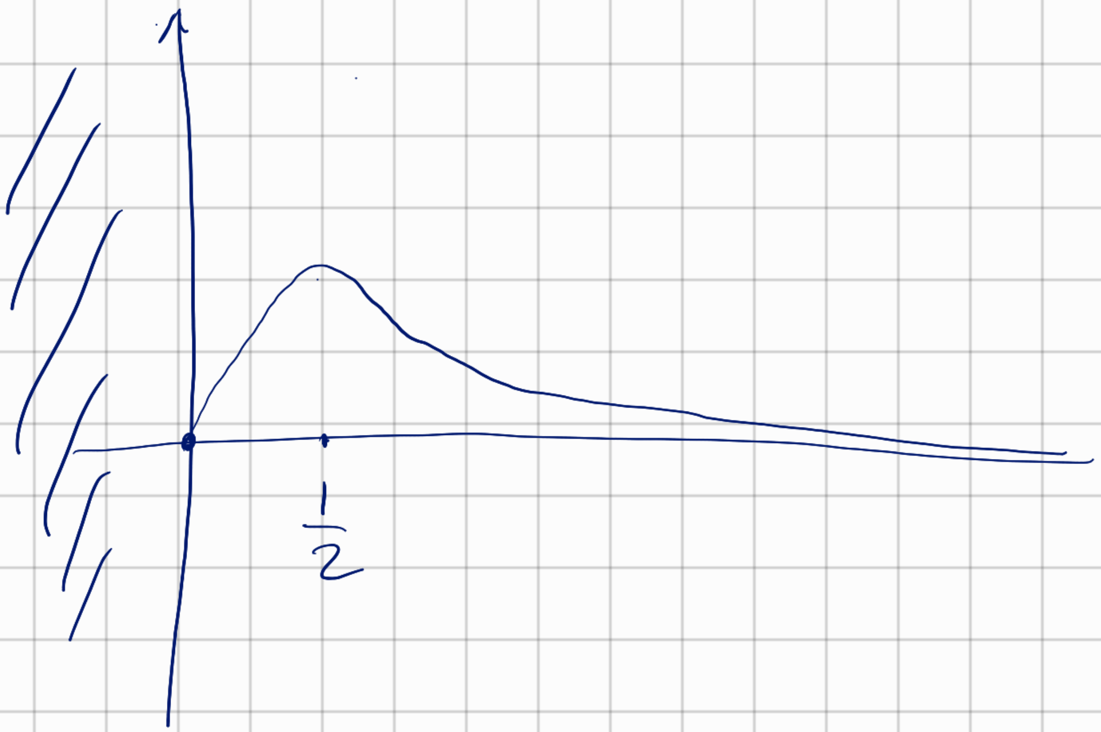
this is
a local max

There is just one extreme point which is a local max

$$\lim_{x \to +\infty} \sqrt{x} \cdot e^{-x} = \lim_{x \to +\infty} \frac{\sqrt{x}}{e^{x}} = \frac{+\infty}{+\infty}$$

we can use Hopital

$$\overset{H}{=} \lim_{x \to +\infty} \frac{1}{2\sqrt{x}\, e^{x}} = 0$$



$$f(x) = \frac{(4+x)^2}{x-1}$$

Domain $\{x \in \mathbb{R} \,/\, x \neq 1\}$

$$f'(x) = \frac{2(x+4)(x-1) - (x+4)^2}{(x-1)^2}$$

$$= \frac{(x+4)(2x-2-x-4)}{(x-1)^2}$$

$$= \frac{(x+4)(x-6)}{(x-1)^2}$$

$f'(x) = 0$ if $x = -4, +6$

$f'(x) > 0$ $x < -4$ and $x > 6$

		-4		6	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	MAX	$\searrow$	MIN	$\nearrow$

The function is increasing when
$x \leq -4$ and $x \geq 0$
decreasing when $-4 \leq x \leq 6$
We have 2 critical pts
$x = -4$ local max
$x = 6$ local min

$$\lim_{x \to +\infty} \frac{f(x)}{2x} = \lim_{x \to +\infty} \frac{x^2 + \text{lower deg terms}}{2x^2 + \text{lower deg}}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^2}}{2\cancel{x^2}} = \frac{1}{2}$$

The function has no global max & min since when x goes to 1 it can be arbitrarily big or small

$f(x) = x^3(x-1)$

$f'(x) = 3x^2(x-1) + x^3$

$= 4x^3 - 3x^2$

$= \underbrace{x^2}_{\geq 0}(4x-3)$

The derivative is ≥ 0

when $4x - 3 \geq 0$

$x \geq \frac{3}{4}$

$f'(x) = 0$ in $x=0$ and

$x = 3/4$

Thus the function is increasin

when $x \geq 3/4$ & decrease

when $x \leq 3/4$

		0		$3/4$	
$f'(x)$	$-$	0	$-$	0	$+$
$f(x)$	$\searrow$		$\searrow$		$\nearrow$
		inflectio point		local min	

in $[-1, 1]$ there are 2 critical points 0 and $3/4$

Candidates for max & min

Extremes

-1 MAX $\boxed{f(-1) = 2}$

1 $f(1) = 0$

Critical pts

0 $f(0) = 0$

$3/4$ MIN $\boxed{f(3/4) = \frac{27}{56}\left(-\frac{1}{4}\right)} < 0$

In $[-1, 1]$ the function max value is 2 taken in

$$x = -1$$

and the function min value is $-\frac{27}{216}$ taken in

$$x = \frac{3}{4}$$

$$\lim_{x \to +\infty} \frac{x^3(x-1)}{e^x} =$$

$$\lim_{x \to +\infty} \frac{x^4}{e^x} + \cancel{\frac{-1/x}{e^x}} \to 0 =$$

$$= \lim_{x \to +\infty} \frac{x^4}{e^x} = \frac{+\infty}{+\infty}$$

I can use l'Hôpital

$$\stackrel{H}{=} \lim_{x \to +\infty} \frac{4x^3}{e^x}$$

$$\stackrel{H}{=} \lim_{x \to +\infty} \frac{12x^2}{e^x}$$

$$\stackrel{H}{=} \lim_{x \to +\infty} \frac{24x}{e^x}$$

$$\stackrel{H}{=} \lim_{x \to +\infty} \frac{24}{e^x} = 0$$