Solution to Slides lecture 9

$$\int (3\sqrt{x} + 6x^2 - x^{-1} + e^x)\, dx =$$

$$= \int 3\sqrt{x}\, dx + \int 6x^2 dx - \int x^{-1} dx + \int e^x dx$$

$$= \int 3x^{1/2} dx + \frac{6}{3}x^3 + C_1 - \ln|x| + C_2 + e^x + C_3$$

$$= \frac{3}{\frac{3}{2}} x^{3/2} + \frac{6}{3}x^3 - \ln|x| + e^x + C$$

$$= 2x^{3/2} + 2x^3 - \ln|x| + e^x + C$$

$$\int x e^x dx = xe^x - \int \frac{d}{dx}x \cdot e^x dx$$

$$= xe^x - \int e^x dx = e^x(x-1) + C$$

$$\int \ln(x) dx = \int 1 \cdot \ln(x) dx = x\ln(x) - \int x \frac{d}{dx} \ln x\, dx$$

$$= x\ln(x) - \int x \frac{1}{x} dx =$$

$$= x\ln(x) - \int 1\,dx$$

$$= x\,(\ln(x) - 1) + C$$

$$\int \frac{t\ln(t^2+1)}{t^2+1}\,dt$$

$u = t^2+1$

$\frac{du}{dt} = 2t$

$du = 2t\,dt$

$dt = \frac{du}{2t}$

$$= \int \frac{\cancel{t}\ln(u)}{u}\,\frac{du}{2\cancel{t}}$$

$$= \int \frac{\ln(u)}{u}\,du =$$

$v = \ln(u)$

$\frac{dv}{du} = \frac{1}{u}$

$= du = u\,dv$

$$= \int \frac{v}{\cancel{u}}\,d\cancel{x}\,\cancel{u}$$

$$= \frac{1}{2}v^2 + C$$

$$= \frac{1}{2}\ln(u)^2 + C$$

$$= \frac{1}{2}\ln(t^2+1) + C$$

$$\int \ln(1+2t)\left(\frac{1}{2}+t\right)^2 dt$$

$u = \frac{1}{2} + t \qquad t = u - \frac{1}{2}$

$du = dt$

$$= \int \ln(\cancel{1}+2u-\cancel{1})\, u^2\, du \quad = \text{by parts}$$

$$= \frac{1}{3}u^3 \ln(2u) - \int \frac{1}{3}u^{\cancel{3}\,2} \cdot 2\, \frac{1}{\cancel{u}}\, du$$

$$= \frac{1}{3}\left(1/2+t\right)^3 \ln(1+2t) - \int \frac{2}{3}u^2\, du$$

$$= \frac{1}{3}\left(1/2+t\right)^3 \ln(1+2t) - \frac{2}{9}\left(\frac{1}{2}+t\right)^3 + C$$

$$\int (9-x^3)x^2\, dx$$

$u = 9 - x^3$

$du = 3x^2\, dx$

$dx = du/3x^2$

$$= \int u \, \cancel{x^2} \frac{du}{3\cancel{x^2}} = \int \frac{u}{3} du$$

$$= \frac{1}{6} u^2 + C$$

$$= \frac{1}{6} (9 - x^3) + C$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$dx = 2\sqrt{x} \, du$

$$\int \frac{e^u}{\cancel{\sqrt{x}}} 2\cancel{\sqrt{x}} \, du = \int 2e^u du$$

$$= 2e^u + C = 2e^{\sqrt{x}} + C$$

Example

$$\frac{d}{dx}\int_0^{x^2} e^s \, ds$$

let $F(x)$ a primitive of e^x. $F(x) = e^x$

$$\frac{d}{dx}\int_0^{x^2} e^s \, ds = \frac{d}{dx}\left(F(x^2) - F(0)\right)$$

$$= \frac{d}{dx} F(x^2) - \cancel{\frac{d}{dx} F(0)}$$

$$= 2x\, F'(x^2)$$

$F'(x) = e^x$

$$= 2x\, e^{x^2}$$

$$\int_6^{49} \frac{1}{2x-5} \, dx$$

$u = 2x - 5$

$du = 2\,dx$

$x = 6 \quad u = 7$

$x = 49 \quad u = 93$

$$= \int_7^{93} \frac{1}{u} \frac{du}{2} =$$

$$= \frac{1}{2} \int_7^{93} \frac{1}{u} du$$

$$= \frac{1}{2} \left[\ln|u| \right]_7^{93} =$$

$$= \frac{1}{2}\left(\ln(93) - \ln(7)\right)$$

$$= \frac{1}{2} \ln\left(\frac{93}{7}\right)$$

$$\int_{-1}^{1} \frac{x^2}{2} dx = \left[\frac{1}{6} x^3 \right]_{-1}^{1}$$

$$= \left[\frac{1}{6} 1^3 - \frac{1}{6}(-1)^3 \right)$$

$$= \frac{2}{6} = \frac{1}{3}$$

$$\int_1^{e^2} x^3 \ln x \, dx$$

$$= \left[\frac{1}{4} x^4 \ln x\right]_1^{e^2} - \int_1^{e^2} \frac{1}{4} x^{4\,3} \frac{1}{\cancel{x}} \, dx$$

$$= \frac{1}{4} e^8 \cdot \ln(e^2) - \cancel{\frac{1}{4} 1^4 \cdot \ln(1)}$$

$$- \left[\frac{1}{16} x^4\right]_1^{e^2}$$

$$= \frac{1}{2} e^8 - \frac{1}{16} e^8 + \frac{1}{16}$$

$$= \frac{7}{16} e^8 + \frac{1}{16}$$

$$\int_1^{+\infty} x^2\,dx = \lim_{a\to+\infty} \int_1^a x^2\,dx$$

$$= \lim_{a\to+\infty} \left[\frac{1}{3}x^3\right]_1^a$$

$$= \lim_{a\to+\infty} \frac{1}{3}a^3 - \frac{1}{3} = +\infty$$

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{a\to+\infty} \int_1^a \frac{1}{x}\,dx$$

$$= \lim_{a\to+\infty} \left[\ln|x|\right]_1^a$$

$$= \lim_{a\to+\infty} \ln(a) - \cancel{\ln(1)} = +\infty$$

$$\int_0^{+\infty} e^{-x}\,dx = \lim_{a\to+\infty} \int_0^a e^{-x}\,dx$$

$$= \lim_{a\to+\infty} \left(-e^{-x}\right)_0^a =$$

$$= \lim_{a \to +\infty} \left(\cancel{-e^{-a}}^{\,\to 0} + e^{-0} \right)$$

$$= 1$$

$$\int_0^1 \frac{1}{\sqrt{x}}\, dx$$ notaa of domain $\frac{1}{\sqrt{x}}$

$$= \lim_{b \to 0^+} \int_b^1 \frac{1}{\sqrt{x}}\, dx$$

$$= \lim_{b \to 0^+} \left[\frac{1}{-\frac{1}{2}+1}\, x^{1/2} \right]_b^1$$

$$= \lim_{x \to 0^+} \left[2\sqrt{x} \right]_b^1$$

$$= \lim_{x \to 0^+} 2\sqrt{1} - 2\sqrt{b} = 2$$