Solution to slides lecture 10/11

$f(x,y) = x^2 + y^2$ the domain is the whole $\mathbb{R}^2$

$f(x,y) = x^2 - y^2$ the domain is the whale $\mathbb{R}^2$

$f(x,y) = x^2 e^{x+y}$ the domain is the whole $\mathbb{R}^2$

$$f(x,y) = \frac{1}{x^2+2y^2+1}$$

we need $x^2 + 2y^2 + 1 \neq 0$

but this is always ≥ 1

$\Rightarrow$ the domain is the whole $\mathbb{R}^2$

$$\frac{\partial}{\partial y}(x^2+y)e^{xy}$$

$$= 1 \cdot e^{xy} + (x^2+y) \cdot x \cdot e^{xy}$$

$$\frac{\partial}{\partial y \partial y} x^3 e^{y^2} = \frac{\partial}{\partial y} 2yx^2 e^{y^2}$$

$$= 2x^2 e^{y^2} + 2yx^2 \cdot 2y e^{y^2}$$

$$= 2x^2 e^{y^2} + 4y^2 x^2 e^{y^2}$$

$f(x,y) = x^2 y + 3y$ $\qquad x = t^2$

$\qquad\qquad\qquad\qquad y = t^3$

$$\frac{\partial}{\partial x} f(xy) = 2xy$$

$$\frac{\partial}{\partial y} f(xy) = x^2 + 3$$

$$g(t) = f(x(t), y(t))$$

$$\frac{d}{dt} g(t) =$$

$$\frac{\partial}{\partial x} f(x(t)\, y(t)) \frac{d}{dt} x(t) +$$

$$\frac{\partial}{\partial y} f(x(t)\, y(t)) \frac{d}{dt} y(t)$$

$$= \left(2x(t) y(t) \right) 2t +$$

$$\left(x(t)^2 + 3 \right) 3t^2$$

$$= 2t^2 \cdot t^3 \cdot 2t + \left(t^4 + 3 \right) 3t^2$$

$$= 9t^2 + 3t^6 + 4t^6$$

$$= 9t^2 + 7t^6$$

Sanity check:

In this case we can

compute $g(t)$ explicitly

$$g(t) = t^4 \cdot t^3 + 3t^3$$

$$= t^7 + 3t^3$$

$$\frac{d}{dt} g(t) = 7t^6 + 9t^2$$