Solution to slides 14

$$\begin{cases} x + 2y - 3z = 4 \\ 3x - y + 5z = 2 \\ 4x + y \quad (c^2-14)z = c+2 \end{cases}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & c^2-14 & c+2 \end{array}\right] \quad \begin{array}{l} R_2 - 3R_1 \\ R_3 - 4R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & \boxed{-7} & 14 & -10 \\ 0 & -7 & c^2-2 & c-14 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & c^2-16 & c+4 \end{array}\right]$$

just one solution if

$c^2-16 \neq 0 \qquad c = \pm 4$

if $c=4$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & -7 & 14 & 0 \\ 0 & 0 & 0 & 8 \end{array}\right]$$ No sol

$c=-4$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & -7 & 14 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

3 variables $\qquad$ 2 rows $\neq 0$

$3-2=1>0 \qquad \infty$ many sol.

$$\begin{bmatrix} 2 & 0 & 2+k \\ 3 & 1 & 0 \\ k & 0 & -2 \end{bmatrix}$$

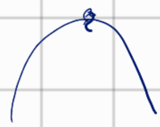
$$\det(A) = 1 \det \begin{pmatrix} 2 & 2+k \\ k & -2 \end{pmatrix}$$

$$= -4 - k(2+k)$$

$$= -k^2 - 2k - 4$$

$$= -(k^2 + 2k + 4) = 0$$

$$\Leftrightarrow k^2 + 2k + 4 = 0$$



$$\Delta = 4 - 16 < 0$$

this has no solutions

So $\det(A) \neq 0$ for all k

which means that A is invertible

$\det(A)' = -2k - 2 = 0$

$k = -1 \rightsquigarrow -(1-2+4) = -3$

$k = -2 \rightsquigarrow -(4-4+4) = -4$

$k = 2 \rightsquigarrow -(4+4+4) = -16$

the max is -3 and the min is -16