Stockholm University

Mathematical Methods for Economists

Lecture 1:Review of elementary algebra

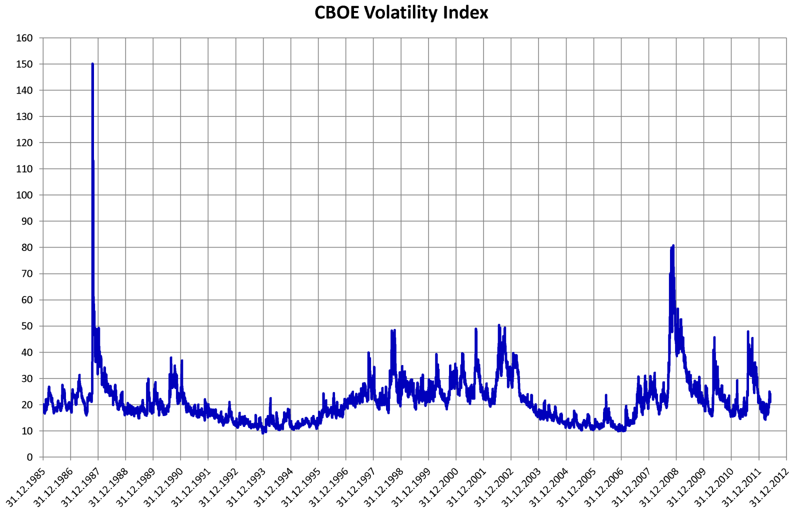
Sofia Tirabassi

Logistic Information

- **Book:** Sydsaeter & Hammond, *Essential Mathematics for Economic Analysis (6:th ed)*
- **Final Exam: 2026-03-16, 14:00-19:00**
- **Homework:** Two sets, prepares for the final. Details may be found on course page.

Disclaimer

We are not going to do this:



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Because you would need to know this:

Definition of VIX

VIX expresses volatility in percentage points. It is calculated as 100 times the square root of the expected 30-day variance of the rate of return of the forward price of the S&P 500 index.

$$\text{VIX} = 100\sqrt{\text{forward price of realized cumulative variance}}$$

Suppose the forward price F_t of the S&P index under a risk neutral measure Q follows

$$\frac{dF_t}{F_t} = \sigma_t \; dW_t \text{ so that } d\ln F_t = -\frac{\sigma_t^2}{2}\; dt + \sigma_t \; dW_t.$$

Here, the instantaneous volatility function σ_t is stochastic.

When one computes the differential of a function of F_t, an additional drift term $-\frac{\sigma_t^2}{2}dt$ in $d\ln F_t$ arises from the Ito lemma, where

$$\frac{1}{2}\sigma_t^2 F_t^2 (dW_t)^2 \frac{\partial^2}{\partial F_t^2}\ln F_t = -\frac{\sigma_t^2}{2}\; dt.$$

So what are we going to do?

- Preparatory algebra
- Geometric sums.
- Functions of one variable.
- Derivatives, integration.
- Optimization.
- Same for functions in two variables.
- Linear algebra: Systems of linear equations. Determinants.

It is a lot of material and the pace will be quick. This class cover part of the mathematical background that prestigious schools think you need in order to enter their programs:

`https://www.imperial.ac.uk/business-school/programmes/msc-risk-management/admissions/self-assessment-maths-test/`

Section 1
Lecture 1

Lecture Goal and Outcome

Goal: Review of elementary algebra (should be at the Matte 2b/c level)

Learning Outcome: At the end of the lecture you will be able to give meaning to expressions of the form

$$a^q,$$

where a is a positive real number and q is any rational number.

Study Tip: This should be a review of topic you encountered in your high school math classes (Should be Matte 2 b or c). Skim through chapter 2 of the book and try the difficult exercises. If you have problem read more carefully the text and exercise more.

Why you should care

Powers appears in many instance in Economics:

$$K_t = K_0 \left(1 + \frac{p}{100}\right)^t$$

Need for negative exponents: How much should I invest today if I want K kr when I retire in 40 years?

Need for rational exponents: How much money will I have in 30 months?

Why you should care II

Here is a recurring formula appearing in studies of national economic growth:

$$Y = 2{,}262(K)^{0{,}203}(L)^{0{,}763}(1{,}02)^{t}$$

where

- Y is the national product
- K is the capital invested in stock
- L is the labor
- t is time

Lecture Plan

- Numbers Sets (natural numbers, integers, rational and real numbers). Operations and rules. Operation with fractions.
- Powers (with rational and integer exponents). Rules for computations.
- Algebraic expressions. Computing and simplifying algebraic expressions.
- Binomial coefficients.

Section 2
Numbers Sets

Number Sets in Economic Modeling

We classify numbers based on how they appear in economic data:

- **Natural Numbers** ($\mathbb{N}$): $\{1, 2, 3, \dots\} \rightarrow$ Discrete quantities (e.g., number of factories).
- **Integers** ($\mathbb{Z}$): $\{\dots, -2, -1, 0, 1, 2, \dots\} \rightarrow$ Deficits, absolute returns.
- **Rational Numbers** ($\mathbb{Q}$): Fractions $\frac{a}{b} \rightarrow$ Exact prices, market shares ($p = \$19{,}50$, $ms = \frac{1}{4}$).
- **Real Numbers** ($\mathbb{R}$): Continuous spectrum $\rightarrow$ Essential for continuous compounding interest (e^r).

Rules of operations

- $a + b = b + a$
- $ab = ba$
- $(a + b) + c = a + (b + c)$
- $(ab)c = a(bc)$
- $a(b + c) = ab + ac$
- $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
- $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
- $\frac{a}{b} = \frac{ac}{bc}$ for $c \neq 0$
- $\frac{a}{b} / \frac{c}{d} = \frac{ad}{bc}$ for $c \neq 0$

Examples (5-6 minutes)

- $\frac{1}{2} + 1 + \frac{1}{3} + \frac{1}{4} =$
- $\left(\frac{1}{2} - \frac{1}{3}\right) / \left(\frac{3}{4} - \frac{1}{2}\right) =$
- Simplify $\frac{7}{28}$
- Simplify $\frac{153}{42}$

1. **Think:** Work on your own for 2 minutes.
2. **Pair:** Compare your strategy with your neighbor.
3. **Share:** Let's look at the answer together.

Questions?

Section 3
Powers

Rules for Powers with

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Fundamental Laws:

- $a^m \cdot a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{m \cdot n}$
- $(ab)^n = a^n b^n$

Negative & Fractional Exponents:

- $a^{-n} = \frac{1}{a^n}$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Economic Context: If a market expands by 5% per year, the growth multiplier over a single month is exactly $1{,}05^{\frac{1}{12}}$.

Common misconception

$$\sqrt{4} = \pm 2$$

This is false! We have that $\sqrt{4} = 2$. However we have that $x^2 = 4$ has two solutions: $\pm\sqrt{4} = \pm 2$,

Application: Cobb-Douglas Production Function

Macroeconomics evaluates aggregate production using fractional powers:

$$Y = AK^{\alpha}L^{\beta}$$

- $Y =$ Total Output
- $K =$ Capital Input, $L =$ Labor Input
- $\alpha, \beta =$ Output Elasticities (often $\alpha + \beta = 1$)

To evaluate marginal changes, you must seamlessly shift exponents up and down using the rules of powers.

Active Learning 2: Error Analysis (7 Mins)

Spot the Analytics Mistake

A junior economist tries to scale up a production function block in a report and writes:

$$(2K^2 \cdot L^3)^3 = 2K^6L^9$$

Discuss with your group:

- What rule did the economist forget to apply?
- What is the correct mathematical expression?

Examples (5 minutes each)

Compute the following expressions:

$$\frac{2^{\frac{1}{4}} \cdot 8^{\frac{1}{4}} \cdot 4^{\frac{1}{4}}}{2 \cdot 2^{-\frac{1}{2}}} =$$

$$3\left(\sqrt{3} + 2\sqrt{\frac{1}{3}}\right)^2 =$$

Questions?

Section 4
Algebraic expressions

Algebraic expressions

An algebraic expression is anything that can be created with letters ($a, b, c \cdots x, y, z$) operations sign (=+-/$\times$) powers and real numbers.

- $E = mc^2$
- $K_t = K_0(1 + \frac{p}{100})^t$
- $x^2 + y^2 + b$

You can do operation with algebraic expressions and the same rule for computing holds as for operation between numbers.

Examples (5 minutes for 1)

- $(a+b)^2 = a^2 + 2ab + b^2$
- $a^2 - b^2 = (a+b)(a-b)$
- Expand $(1 - y^{-2})(y^2 - 2y^{-1} + 3)$
- Compute $(1+\sqrt{7})(1-\sqrt{7})$
- Simplify

$$\frac{8\sqrt[3]{x^2}\sqrt[4]{y}\sqrt{\frac{1}{z}}}{-2\sqrt[3]{x}\sqrt[4]{y^5}\sqrt{z}}$$

- Factor $e^{3x} + 2e^{2x} + e^{x}$
- Factor $7x^3y - 28xy$
- Simplify

$$\frac{30x^2 - 30a^2}{x+a}$$

Questions?

Section 5
Binomial Coefficients

Factorial

Let n a natural number, we define the factorial of n as:

$$n! = 1 \cdot 2 \cdots (n-1) \cdot n \quad \textbf{if } n > 0$$

$$0! = 1$$

Binomial coefficients

Given n and k two natural numbers the binomial coefficient "n choose k" is denoted by

$$\binom{n}{k}$$

and compute the number of ways we can pick k distinct object out of n (the order does not count).

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{if } k \leq n$$

$$\binom{n}{k} = 0 \quad \text{if } k > n$$

Active Learning: Quick Portfolio Design

Problem

A portfolio manager selects exactly 2 tech companies out of a pool of 5 total candidates to hedge risk. How many distinct asset combinations are possible?

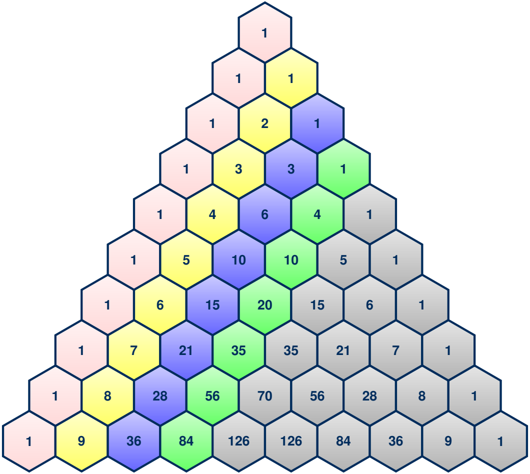
Use the formula for $\binom{n}{k}$ where $n = 5$ and $k = 2$. Take 2 minutes to calculate.

Useful Identities

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

Pascal Triangle



Newton binomial theorem

Binomial Theorem

Given n a natural number we have that

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{1}b^n$$
$$= \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

- $(a+b)^2 =$
- $(a+b)^5 =$

Thank you for your attention!