Nonlinear Algebra

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KTH - Royal Institute of Technology

September 2, 2026

Important!

- **This class relies on active participation & learning by doing!** We might bring you outside your comfort zone, by grouping you with people from different backgrouns/programs. The goal is to **have fun, while challenging ourselves.**
  - $\rightarrow$ come to every lecture and, most of all, exercise session!

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  - $\rightarrow$ bonus points: $\lceil \frac{\#\text{attendances}}{15} \cdot 10 \rceil$

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- The first lecture will happen next week: check out the slides if you are curious! Today is a warm-up / crash course on algebraic varieties.
  - → If some of today's problems seem too hard or boring, don't get discouraged: during the next few weeks it will become clear why we need those things and we'll manage together :)

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30% of the total score will be obtained in 2 labs: The first lab happen on Wednesday, September 30, 2026, 9:00-12:00.

TODOs for now

We'll give you more exercises than can be discussed in exercise sessions. We strongly encourage you to write (partial) solutions for some of the other exercises and share them with your peers and with us for feedback!

- **Vote for a platform that facilitates this:**
  1. Slack
  2. Discord
  3. Discussion forum on `kurser.math.su.se`
  4. something else?

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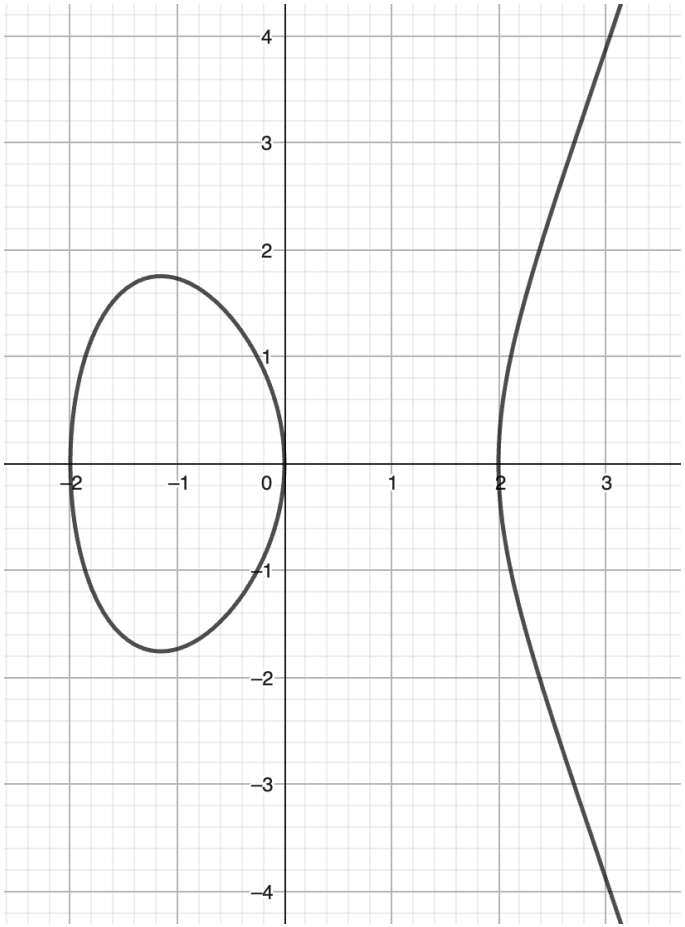
- Find the course *Nonlinear Algebra* on `kurser.math.su.se` and register as a participant

  $\rightarrow$ all grades and announcements will be communicated there!
- Fill out the participant table from today

  $\rightarrow$ we will compare this list with the online registrations

Curves



$$y^2 = x^3 - 4x$$

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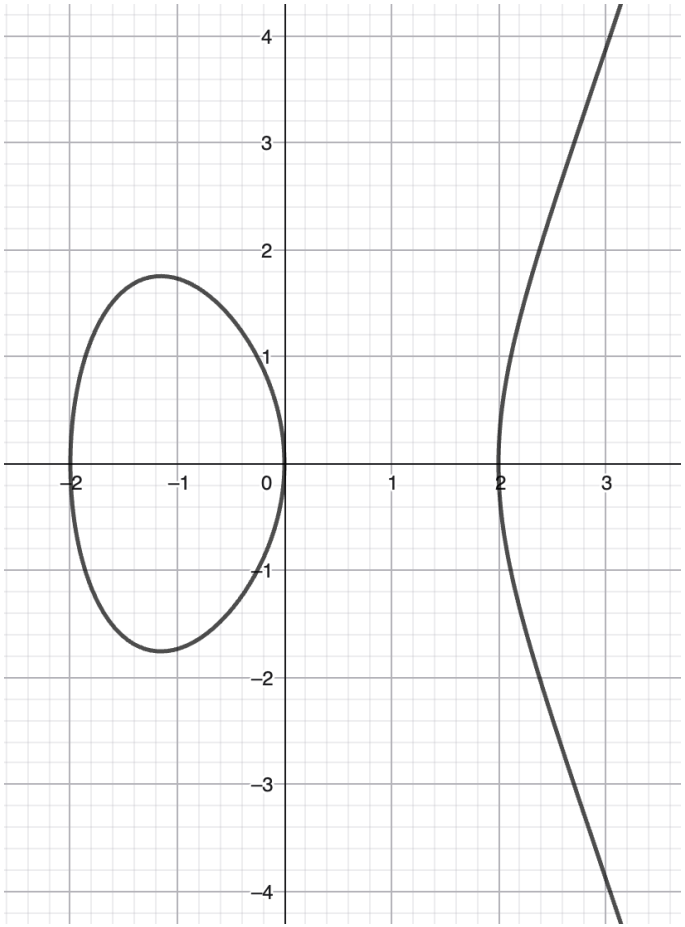

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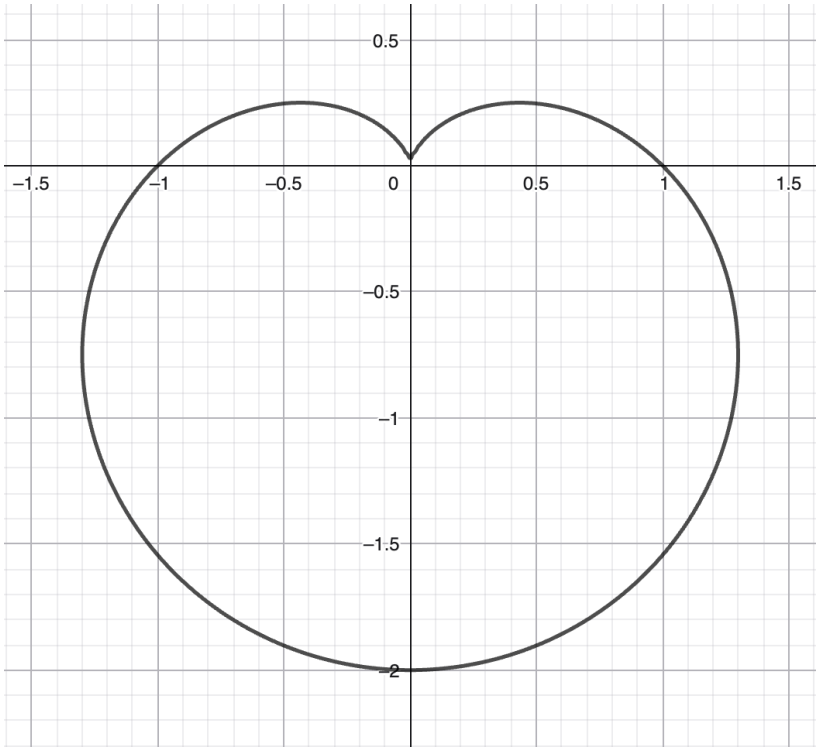
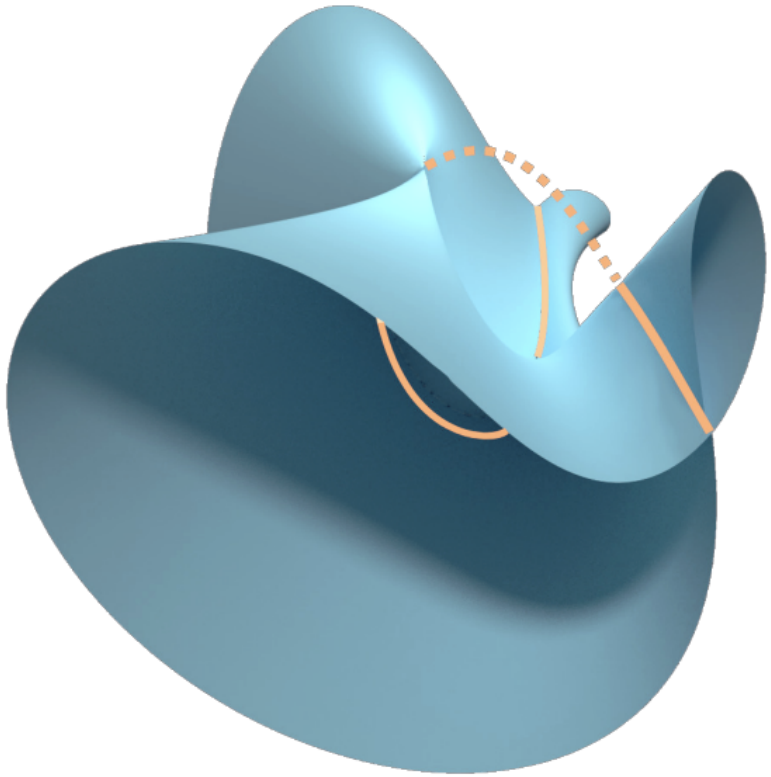


$$0 = x^4 + 2x^2y^2 + y^4 + 2y^3 + 2yx^2 - x^2$$

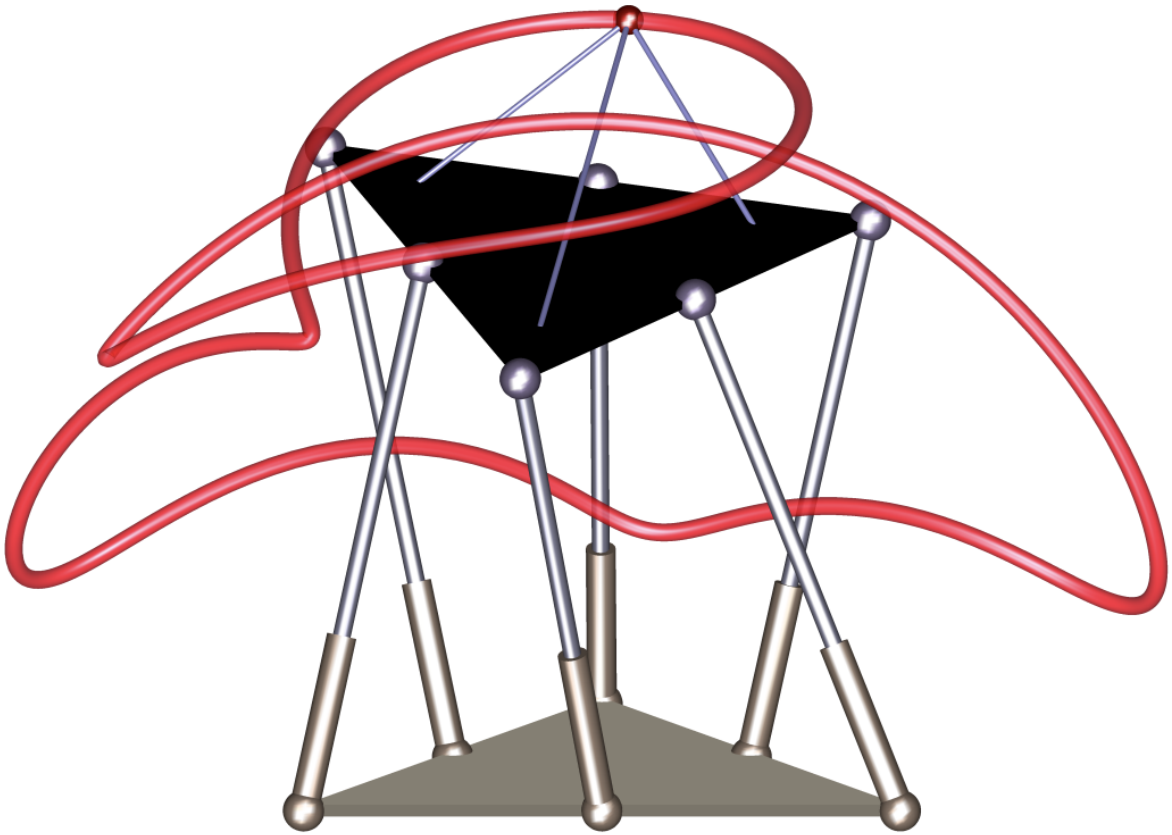
Functions parametrized by a Lightning Self-Attention Mechanism



Baby ChatGPT [1]

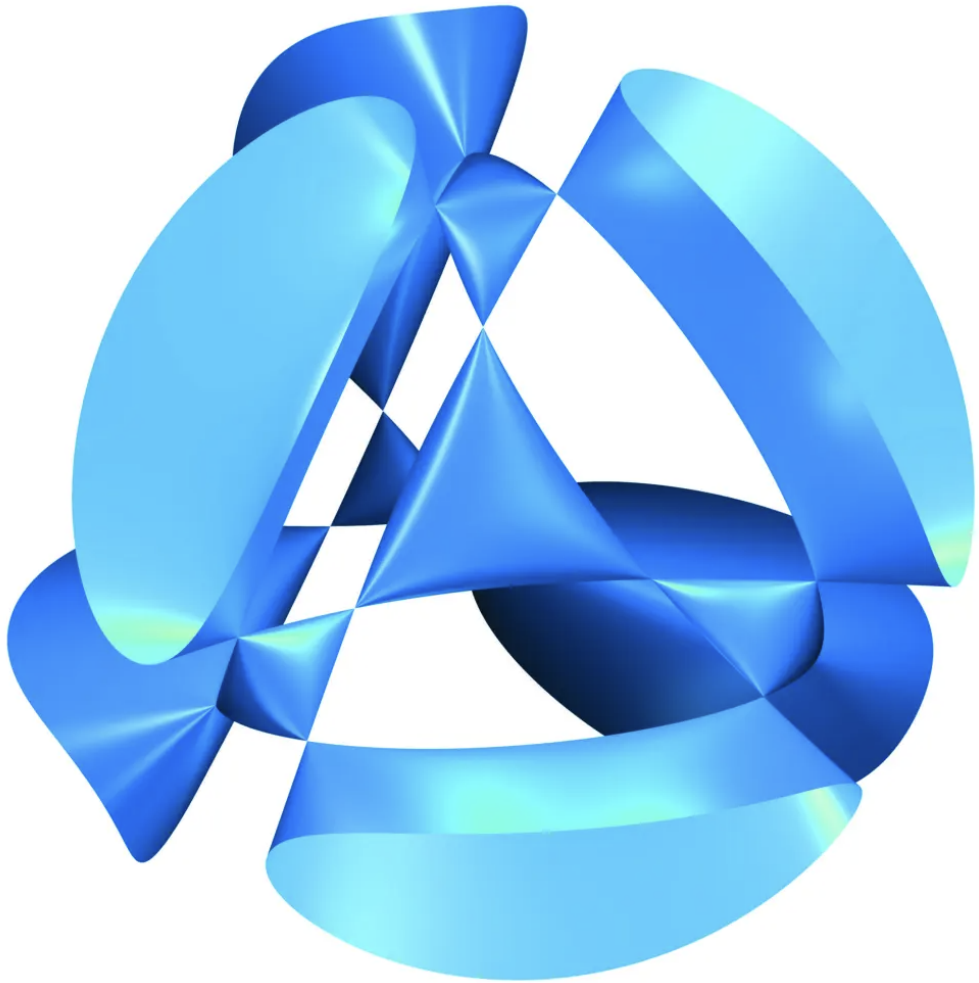
$$0 = c_1^2 c_6^2 + c_3^2 c_4^2 + c_1 c_3 c_5^2 + c_2^2 c_4 c_6 - 2 c_1 c_3 c_4 c_6 - c_1 c_2 c_5 c_6 - c_2 c_3 c_4 c_5$$

Griffis-Duffy Type I Platform



[2]

Kummer-Surface with $\mu = \sqrt{2}$



$$0 = (x^2 + y^2 + z^2 - 2)^2 - 5((z-1)^2 - 2x^2)((z+1)^2 - 2y^2) \quad [2]$$

References


Marchetti, G. L.; Connelly, E.; Paul Breiding, P.; Kohn, K. (2025): Critical Points of Degenerate Metrics on Algebraic Varieties: A Tale of Overparametrization


Seigal, A. (2016): Picture this maths, https://picturethismaths.wordpress.com/2016/02/22/siaga-introduction/