MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Boris Shapiro

Tentamensskrivning i
Matematik 3, kombinatorik, 7,5 poäng
den 13 januari 2021

11 points will be enough to pass the written exam. The time for the oral examination (to be held on Friday, January 15) for the students who succeeded with the written exam will be announced on Thursday evening, January 14. Good luck!

1. Consider the positive integer 10.

   a) How many partitions of 10 into parts not exceeding 3 exist? 1 p

   b) How many partitions of 10 into at most 3 parts exist? 1 p

   c) How many partitions of 10 in odd parts exist? 1 p

   d) How many partitions of 10 in even parts exist? 1 p

2. How many strings of length n consisting of the digits $\{0, 1, 2\}$ contain an odd number of even digits? 3 p

3. Let C be the board obtained from the 4×4 square by removing the central 2×2 square. (Thus C consists of 12 remaining unit squares.) Calculate the rook polynomial of C. 3 p

4. Let $a(n)$ denote the number of presentations of a positive integer n in the form $n = n_1 + n_2 + \cdots + n_\ell$ where each entry n_j is either 1 or 2 and the order of entries is important. Determine $a(n)$ using an appropriate recurrence relation. 3 p

5. An (undirected) graph G is called *chordal* if every cycle $[a_1, a_2, ..., a_n]$, $a_n = a_1$, $n \geq 4$ in G admits a chord, i.e., there exists an edge (a_i, a_j) between a pair of vertices which are not neighbors in the cycle.

   a) Find a chordal graph with 5 vertices having an Euler circuit, but not a Hamilton cycle. 2 p

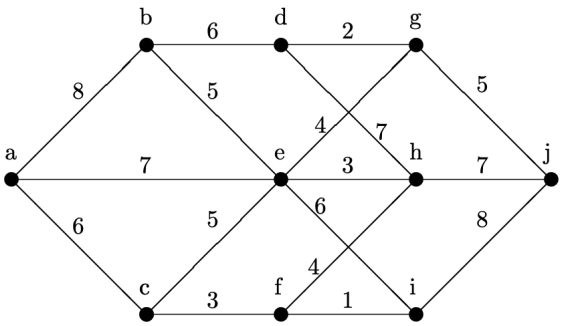
   b) Find a chordal graph with 5 vertices having a Hamilton cycle, but not an Euler circuit. 2 p

6. Consider a weighted graph shown in Fig. 1 below.

   a) Determine its minimal spanning tree. (There might, in principle, exist several minimal spanning trees, but their total weights should be the same). 1 p

   b) Determine a shortest path from the vertex a to the vertex j in the graph. 1 p

   c) Consider this graph as a traffic network with all edges directed from left to right. Determine a minimal cut in this network and try to find a maximal flow from the source a to the sink j. 2 p



Figur 1: Weighted graph for Problem 6. To get a traffic network orient its edges from left to right. (Two pairs of crossing edges in the picture whose crossing points are not labelled do not intersect in the graph).