MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Boris Shapiro

Tentamensskrivning i
Matematik 3, kombinatorik, 7,5 poäng
den 24 febuari 2021

*11 points will be enough to pass the written exam. The oral examination will be held on Friday, February 26. The time slots for those who passed the written exam will be announced on Thursday, February 25. Good luck!*

1. Considered 11 numbered balls.

   a) In how many ways can these balls be distributed in 3 numbered boxes? (Some boxes can be empty.) 1 p

   b) In how many ways can 11 identical bolls be distributed in 3 numbered boxes? (Some boxes can be empty.) 1 p

   c) Same question as in a), but no box can be empty? 1 p

   d) Same question as in b), but no box can be empty? 1 p

   e) How many partitions of 11 in at most 3 parts exist? 1 p

   f) How many partitions of 11 in exactly 3 parts exist? 1 p

2. Solve the linear recurrence equation $a_n - 3a_{n-1} + 2a_{n-2} = 2^n, a_0 = 1, a_1 = 3$. 3 p

3. The *line* graph $L(G)$ of a graph $G = (V, E)$ is a graph with $E$ as its vertex set and two vertices in $L(G)$ are adjacent if and only if their corresponding edges in $G$ share a vertex. Consider the graph $H$ having the vertex set $\{a, b, c, d, e\}$ and the edge set $\{(a, b), (a, c), (c, d), (c, e), (d, e)\}$.

   a) How many vertices has $L(H)$? 1 p

   b) How many edges has $L(H)$? 1 p

   c) Does $L(H)$ have an Euler circuit? 1 p

   d) Does $L(H)$ have a Hamilton path? 1 p

   e) What is the chromatic number of $L(H)$? 1 p

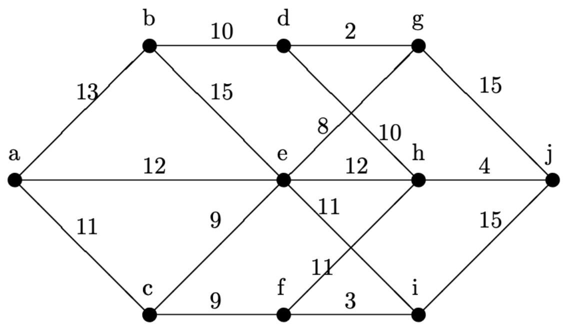
4. How many integer solutions has the equation $x_1 + x_2 + x_3 + x_4 = 8$, subject to the conditions: $x_1 \geq 0$, $x_2 \geq 2$, $2 \leq x_3 \leq 4$, and $x_4$ is even and nonnegative? 3 p

5. Consider a weighted graph shown below.

   a) Determine a minimal spanning tree and its weight. 1 p

   b) Determine a shortest path from the vertex $a$ to the vertex $j$ and its weight in the graph. 1 p

   c) Now consider this graph as a traffic network with all arrows directed from the left to the right. Determine a maximal flow from $a$ to $j$ and a minimal cut. 2 p




Figur 1: Graph for Problem 5