MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
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Lösningar till tentamensskrivning i
Matematik 3, kombinatorik , 7,5 hp
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1. a) We need to count all functions from $\{1, 2, ..., 10, 11\}$ to $\{1, 2, 3\}$. Answer: 3^{11}.

b) Answer: $\binom{13}{2}$

c) We need to count surjections from $\{1, 2, ..., 10, 11\}$ to $\{1, 2, 3\}$. Answer: $3^{11} - 3 \cdot 2^{11} + 3 \cdot 1^{11} = 171006$.

d) Answer: $\binom{13}{2} - 3\binom{12}{1} + 3\binom{11}{0} = 45$.

e) Answer: 16. Count by hand or use the generating function technique.

f) Answer: $16 - 5 - 1 = 10$. (There are 16 partitions in at most 3 parts, 5 in exactly 2 parts, and 1 in one part.)

2. There is a particular solution of the form $cn \cdot 2^n$. Substituted in the equation this gives

$$cn \cdot 2^n - 3c(n-1)2^{n-1} + 2c(n-2)2^{n-2} = 2n$$

implying that $c = 2$. The general solution to the homogeneous equation is $a \cdot 2^n + b$. In the general solution $(a + 2n)2^n + b$ we use the initial conditions $a_0 = 1, a_1 = 3$. This results in the system $a + b = 1$, $(a + 2)2 + b = 3$, which gives $a = -2, b = 3$.

Answer: $a_n = (2n - 2)2^n + 3$

3. The line graph $L(G)$ of a graph $G = (V, E)$ is a graph with E as vertex set and two vertices in $L(G)$ are adjacent if and only if their corresponding edges in G share a vertex. Let H have the vertex set $\{a, b, c, d, e\}$ and the edge set $\{(a, b), (a, c), (c, d), (c, e), (d, e)\}$.

a) Answer: 5 (There are 5 edges in H.)

b) Answer: 6 They are $(ab) - (ac), (ac) - (cd), (ac) - (ce), (cd) - (ce), (ce) - (de), (cd) - (de)$.

c) No, not all degrees are even.

d) Yes, $(ab) - (ac) - (cd) - (de) - (ce)$.

e) 3, (ab), (cd) in one colour, (ac), (de) in one, (ce) in one.

4. Generating function of compositions (ordered partitions) of the form (x_1, x_2, x_3, x_4) satisfying the conditions $x_1 \geq 0$, $x_2 \geq 2$, $2 \leq x_3 \leq 4$ and x_2 is even and nonnegative is given by

$$F(x) = \frac{1}{1-x} \cdot \frac{x^2}{1-x} \cdot (x^2 + x^3 + x^4) \cdot \frac{1}{1-x^2} = x^4 + 3x^5 + 7x^6 + 12x^7 + 19x^8 + 27x^9 + 37x^{10} + \ldots$$

Answer: Coefficient of x^8 is 19.

5. a) Answer: $a - c, b - d, c - e, c - f, d - g, d - h, e - g, f - i, h - j$. Total weight 66.

b) $a - e - h - j$ of weight 28.

c) Answer: Value 28. Minimal cut $\{a, b, c, d, e, f, h\} \cup \{g, i, j\}$.