STOCKHOLM UNIVERSITY
Department of Mathematics
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Examination in
Mathematics for Economic and Statistical Analysis
MM1005, höstterminen 2019, 7.5 ECTS
Wednesday 2 October 2019

Instructions:
- During the exam you **may not** use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers — communicate your reasoning. Use ordinary language where appropriate, not just mathematical symbols.
- Write clearly and legibly.
- Mark your final answer to each question clearly by putting $\boxed{\text{a box}}$ around it.

Grades: There are 7 questions. Each solved problem is awarded up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to difficulty.

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1. Find all solutions to the following systems of equations:

   (a)
   $$\begin{aligned} 3y - z + w &= 1 \\ -x + 2y + 2z &= 0 \\ -x + y - 3z + w &= -1 \end{aligned}$$

   (b)
   $$\begin{aligned} 2A - B - 2C &= -2 \\ 2A + 2B + 4C &= 1 \end{aligned}$$

2. Let $f(x) = \frac{x^2+1}{x+2}$, and $g(x) = \frac{x^2+2}{x+1}$. Find the following limits:

   (a) $\lim_{x\to 0} f(x)$ (b) $\lim_{x\to\infty} f(x) + g(x)$ (c) $\lim_{x\to\infty} f(x) - g(x)$

3. Let A be the matrix $\begin{pmatrix} 2 & 0 & 2+k \\ 3 & -1 & 0 \\ k & 0 & -2 \end{pmatrix}$, depending on a parameter $k \in \mathbb{R}$.

   (a) Find $\det A$, as a function of k.
   (b) Show that A is always invertible, for any value of k.
   (c) Find the maximal and minimal values $\det A$ can take, for k in the range $-2 \leq k \leq 2$.

4. Take $h(x, y)$ to be the function $(xy + y^2)e^x$. Find all stationary ploints of h, and decide whether each one is a maximum, minimum, or saddle point.

5. Find the following integrals:

   (a) $\displaystyle\int (t+1)e^{t-2}dt$ (b) $\displaystyle\int_1^2 (x+1)(3x-3)dx$

6. A factory produces premium quality bowls and plates. Suppose the daily cost (in €) of producing x bowls and y plates is given by

$$C(x, y) = 500 + 3x + 2y + \frac{2}{1000}x^2 + \frac{1}{1000}y^2 + \frac{2}{1000}xy$$

   and suppose that all bowls and plates produced are sold by the factory operators at 5€ per item.

   (a) Give the daily profit function $P(x, y)$.

(b) Show that the only stationary point of $P(x, y)$ is at $x = -500$, $y = 2000$.

(c) In fact, the maximum profit subject to $x, y \geq 0$ is given by $x = 0$, $y = 1500$. For these values of x and y, find the daily marginal cost for each type of item, i.e. the partial derivatives $C_x(0, 1500)$ and $C_y(0, 1500)$. *(You do not need to justify that these values give the maximum profit; just find the marginal costs.)*

7. Consider the curve given by the equation $5x^2 + 5y^2 + 6xy = 4$.

(a) Find the points where this curve intersects the line $x + y = 0$.

(b) Find the equation of the tangent line to this curve at the point $(\frac{1}{2}, \frac{1}{2})$. *(Hint: consider the curve as a level curve of a function $F(x, y)$.)*

GOOD LUCK! — LYCKA TILL!
